FIITJEE – JEE (Main)

PHYSICS, CHEMISTRY & MATHEMATICS

Applicable for Class – XII PASS

PHASE - 1

Time Allotted: 3 Hours **Maximum Marks: 360**

- Do not open this Test Booklet until you are asked to do so.
- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

Important Instructions:

1. Immediately fill in the particulars on this page of the Test Booklet with *Blue / Black Ball Point Pen. Use of pencil is strictly prohibited.*
2. The Answer Sheet is kept inside this Test Booklet. When you are directed to open the Test Booklet, take out the Answer Sheet and fill in the particulars carefully.
3. The test is of **3 hours** duration.
4. The Test Booklet consists of **90** questions. The maximum marks are **360**.
5. There are ***three*** parts in the question paper A, B, C consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each question is allotted **4 (four)** marks for correct response.
6. *Candidates will be awarded marks as stated above in instruction No.5 for correct response of each question.* ***One mark*** *will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet*
7. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 6 above.
8. Use ***Blue / Black Ball Point Pen only*** for writing particulars / marking responses on ***Side-1*** and ***Side-2*** of the Answer Sheet. ***Use of pencil is strictly prohibited.***
9. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. except the Admit Card inside the examination hall / room.
10. On completion of the test, the candidate must hand over the Answer Sheet to the Invigilator on duty in the Room / Hall. ***However, the candidates are allowed to take away this Test Booklet with them.***
11. **Do not fold or make any stray marks on the Answer Sheet.**

Name of the Candidate (in Capital Letters) :____________________________________

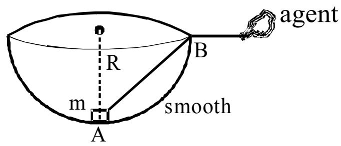
Enrolment Number :____

Batch :________________________ **Date of Examination :** ________________________

(PART – A)

PHYSICS

1. The minimum work done by the agent, in pulling a small particle of mass m from A to B as shown in figure, is



(A) 4 mgR
(B) mgR
(C) 3mgR
(D) 2mgR

2. One end of a spring of force constant k is fixed to a vertical wall and the other to a body of mass m resting on a smooth horizontal surface. There is another wall at a distance x_0 from the body. The spring is then compressed by $2x_0$ and released. The time taken to strike the wall is

(A) $\frac{\pi}{6}\sqrt{\frac{m}{k}}$
(B) $\sqrt{\frac{m}{k}}$
(C) $\frac{2\pi}{3}\sqrt{\frac{m}{k}}$
(D) $\frac{\pi}{4}\sqrt{\frac{m}{k}}$

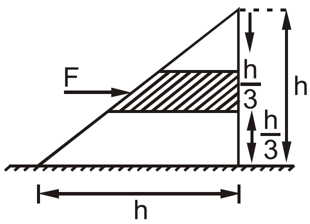
3. A projectile is projected in the earth's gravitational with initial kinetic energy E. The horizontal of the projectile range is R. If the mass of the projectile is 1 kg then the angle of projection of the projectile will be equal to

(A) $\sin^{-1}(gR/2E)$
(B) $2\sin^{-1}(gR/2E)$
(C) $0.5\sin^{-1}(gR/2E)$
(D) $4\sin^{-1}(gR/2E)$

4. A chain of mass 10 kg and length 10 m is resting on a rough horizontal surface (μ = 0.2). A constant force of 20 newton is applied at one end. The tension in the mid-point of the chain is

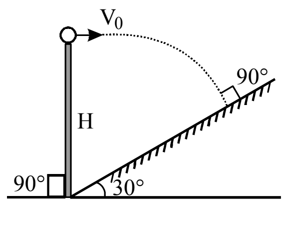
(A) 20 N
(B) 15 N
(C) 10 N
(D) 5 N

5. A wedge of mass *M* resting on a horizontal frictionless surface is given a force *F* in the horizontal direction. The net horizontal force acting on the shaded portion of the wedge is



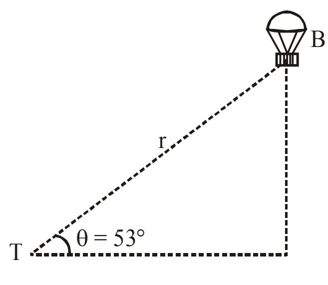
(A) *F*
(B) $\frac{F}{3}$
(C) $\frac{F}{6}$
(D) zero

6. In the given figure, the angle of inclination of the inclined plane is 30°. Find the horizontal velocity V_0 so that the particle hits the inclined plane perpendicularly.



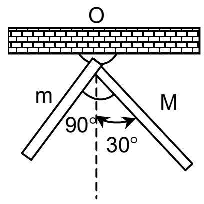
(A) $V_0 = \sqrt{\frac{2gH}{5}}$
(B) $V_0 = \sqrt{\frac{2gH}{7}}$
(C) $V_0 = \sqrt{\frac{gH}{5}}$
(D) $V_0 = \sqrt{\frac{gH}{7}}$

7. A balloon B is moving vertically upward and viewed by a telescope T. At a particular angular position θ = 53° measured parameters are r = 1 km, $\frac{dr}{dt} = 3\,m/s$ and $\frac{d\theta}{dt}$ = 0.002 rad/s. The magnitude of the linear velocity of the balloon at this instant is



(A) 1.2 m/s
(B) 2.4 m/s
(C) 3.6 m/s
(D) 4.8 m/s

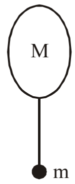
8. Two uniform rods of equal length but different masses are rigidly joined to form an L-shaped body, which is then pivoted as shown in the figure. If in equilibrium the body is in the shown configuration, ratio M/m will be



(A) 2 (B) 3 (C) $\sqrt{2}$ (D) $\sqrt{3}$

9. A particle P suffers an oblique elastic collision with another particle Q at rest initially. If their masses are the same, then after collision
(A) They will move in mutually perpendicular directions
(B) They will move in opposite directions
(C) Particle P continues to move in its original direction while the particle Q remains at rest
(D) Particle Q starts moving in the direction of original motion of P and particle P comes to rest

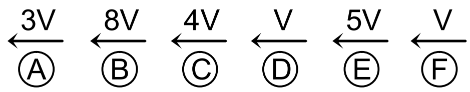
10. A balloon of mass M with a light rope having monkey on the rope is in equilibrium. If the monkey starts moving up with acceleration a w.r.t. rope. Then the acceleration of centre of mass of the system is



(A) $\frac{a(M-m)}{(M+m)}$ (B) Zero (C) a (D) $\frac{aM}{m}$

11. Two particles of masses 1 kg and 3 kg move towards each other under their mutual force of attraction. No other force acts on them. When the relative velocity of approach of the two particles is 2m/s, their centre of mass has a velocity of 0.5 m/s. When the relative velocity of approach becomes 3 m/s, the velocity of the centre of mass is
(A) 0.5 m/s (B) 0.75 m/s (C) 1.25 m/s (D) Zero

12. In the given set of balls all are identical and all the collisions are elastic. Final velocity of ball C will be :



(A) V towards left (B) 5V towards right (C) 3V towards left (D) V towards right

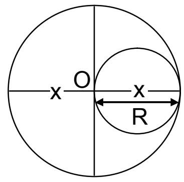
13. The linear density of rod of length L and placed along x–axis with the lighter end at origin, is given by $\lambda = Bx^2$ where B is a constant. Then the co-ordinates of centre of mass are
(A) $\left(\frac{3L}{4},0\right)$ (B) $\left(0,\frac{3L}{4}\right)$ (C) $\left(\frac{4L}{3},0\right)$ (D) $\left(\frac{0,4L}{3}\right)$

14. A particle strikes a horizontal frictionless floor with a speed u, at an angle θ with the vertical and rebounds with a speed v, at an angle φ with the vertical. The coefficient of restitution between the particle and the floor is e. The magnitude of v is
(A) eu (B) $(1-e)u$
(C) $u\sqrt{\sin^2\theta + e^2\cos^2\theta}$ (D) $u\sqrt{e^2\sin^2\theta + \cos^2\theta}$

15. Consider following statements
[1] CM of a uniform semicircular disc of radius R = $2R/\pi$ from the centre
[2] CM of a uniform semicircular ring of radius R = $4R/3\pi$ from the centre
[3] CM of a solid hemisphere of radius R = $4R/3\pi$ from the centre
[4] CM of a hemisphere shell of radius R = R/2 from the centre
Which statements are correct?
(A) 1, 2, 4 (B) 1, 3, 4 (C) 4 only (D) 1, 2 only

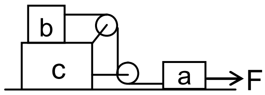
16. A ball falls from a height h_0. There are n collisions with the earth. If after n collisions the ball rises to a height h_n, then coefficient of restitution e is given by:
(A) $e^n = \sqrt{\frac{h_n}{h_0}}$ (B) $e^n = \sqrt{\frac{h_0}{h_n}}$ (C) $ne = \sqrt{\frac{h_n}{h_0}}$ (D) $\sqrt{n}\ e = \sqrt{\frac{h_n}{h_0}}$

17. A spherical hollow is made in a lead sphere of radius R, such that its surface touches the outside surface of lead sphere and passes through the centre. What is the shift in the centre of mass of lead sphere due to the hollowing?



(A) $\frac{R}{7}$ (B) $\frac{R}{14}$ (C) $\frac{R}{2}$ (D) R

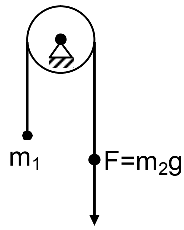
18. Three blocks a, b and c of masses 10 kg, 10 kg and 20 kg are arranged as shown in figure. All the surfaces are frictionless and string is inextensible. Pulleys are light. A constant force F=20N is applied on block a as shown. Pulleys and string are light. Part of the string connecting both pulleys is vertical and part of the strings connecting pulleys with blocks a and b are horizontal.



(A) Acceleration of mass blocks a, b and c is 0.5 m/s^2.
(B) Acceleration of mass block b is 1.
(C) Tension in the string is 10 N.
(D) Acceleration of mass block c is 0.5

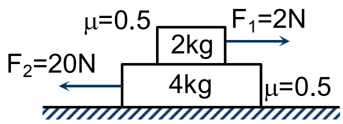
19. A particle starts from rest and travels a distance s with uniform acceleration, then it travels a distance 2s with uniform speed finally it travels a distance 3s with uniform retardation and comes to rest. If the complete motion of the particle is a straight line then the ratio of its average velocity to maximum velocity is:
(A) $\frac{6}{7}$ (B) $\frac{4}{5}$ (C) $\frac{3}{5}$ (D) $\frac{2}{5}$

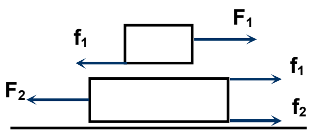
20. A hanging body of mass m_1 is pulled by a force $F = m_2g$ acting on the massless inextensible smooth string. The acceleration of m_1 is:

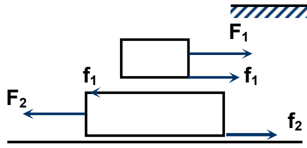


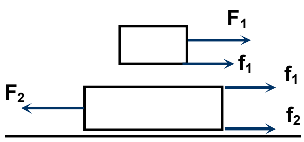
(A) $\frac{m_2 - m_1}{m_1 + m_2}g$ (B) $\frac{m_1}{m_2}g$
(C) $\frac{m_1m_2}{(m_1+m_2)^2 g}$ (D) $\frac{(m_2 - m_1)}{m_1}g$

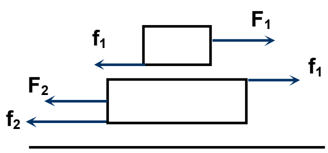
21. The friction force acting between surfaces in contact in the adjoining figure is best represented by



(A)


(B)


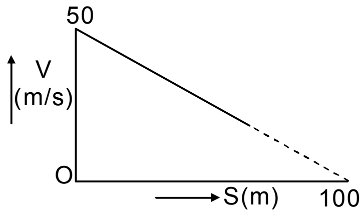
(C)


(D)


22. A ball of mass m is thrown upward with a velocity v. If air exerts an average resisting force F, the velocity with which the ball returns to the thrower is:
(A) $v\sqrt{\frac{mg}{mg+F}}$ (B) $v\sqrt{\frac{F}{mg+F}}$ (C) $v\sqrt{\frac{mg-F}{mg+F}}$ (D) $v\sqrt{\frac{mg+F}{mg-F}}$

23. A block of mass 20 kg is lying on a frictionless table. A block of 5 kg is kept on the block of 20 kg. If a variable force F given by F = kx is applied on the block of mass 20 kg and initially the mass of 20 kg is lying at x=1m and μ = 0.2 and k = 5 N/m, find the distance after which 5 kg mass starts slipping from the starting point?
(A) 8m (B) 3m (C) 9m (D) 6m

24. If the velocity v of particle moving along a straight line decreases linearly with its position. Value a from 50 m/s to a value approaching zero at s=100m, the time it takes to reach 100m position will be:

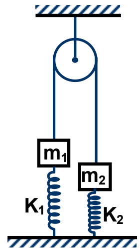


(A) 10s
(B) 5
(C) infinity
(D) 0.5s

25. A particle is projected vertically upward from a point on ground. It takes 2 sec. to reach a point 'A' above ground. It continuous to move and reaches again at same height 4 sec. after crossing A for the first time. The velocity of particle at half the maximum height is:
(A) $10\sqrt{2}$ m/s (B) $20\sqrt{2}$ m/s (C) 10 m/s (D) 20 m/s

26. In a one dimensional collision between two identical particles A and B, B is stationary and A has momentum of magnitude P before impact. During impact, B gives an impulse of magnitude J to A. Then coefficient of restitution between the two is:
(A) $\frac{2J}{P}-1$ (B) $\frac{2J}{P}+1$ (C) $\frac{J}{P}+1$ (D) none of these

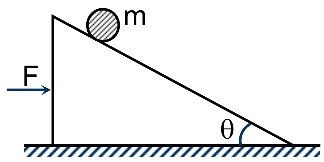
27. The system shown in the figure is in equilibrium. Masses m_1 & m_2 are 2 kg and 8kg respectively. Spring constants k_1 & k_2 are 50N/m and 70N/m respectively. If the compression in second spring is 0.5 m. What is the elongation in first spring? (Both springs have the same natural length)



(A) 1.3 m
(B) 1.0 m
(C) 0.5 m
(D) 0.9 m

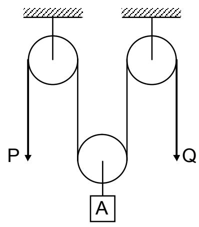
28. A particle moves in a straight line with retardation proportional to its displacement. Its loss of kinetic energy for any displacement x is proportional to:
(A) x (B) x^2 (C) ℓn x (D) e^x

29. In the given figure, the wedge is acted upon by a constant horizontal force F. The wedge is moving on a smooth horizontal surface. A ball of mass 'm' is at rest relative to the wedge. The ratio of forces exerted on 'm' by the wedge when F is acting, and when wedge is kept stationary, assuming no friction between the wedge and the ball, the ratio is equal to



(A) $\sec^2\theta$ (B) $\cos^2\theta$ (C) 1 (D) none of these.

30. In the adjacent figure, the ends P and Q of the string are being pulled down with an acceleration of $3m/s^2$ and $4m/s^2$ respectively. The acceleration of block A will be:



(A) $5m/s^2$
(B) $7m/s^2$
(C) $3.5m/s^2$
(D) $4m/s^2$

space for rough work

(PART – B)

CHEMISTRY

1. How many grams of CaC_2O_4 will dissolve in distilled water to make one litre of saturated solution of it? (K_{sp} for $CaC_2O_4 = 2.5 \times 10^{-9}\ mol^2 lit^{-2}$)
(A) 0.0064 g
(B) 0.1028 g
(C) 0.1280 g
(D) 0.2056 g

2. The time required for 10% completion of a first order reaction at 298 K is equal to that required for is 25% completion at 308 K, if the pre–exponential factor for the reaction is $3.56 \times 10^9\ s^{-1}$. Its energy of activation at 318 K is
(A) 76.37 kJ/mole
(B) 50 kJ/mole
(C) 76.3 J/mole
(D) 50 J/mole

3. At constant temperature the equilibrium constant K_p for decomposition of reaction
$N_2O_4 \rightleftharpoons 2NO_2$ is expressed by $K_p = 4x^2P/1–x$
Which statement is true?
(A) K_p increases with increase of P
(B) K_p increases with decrease of P
(C) K_p increase with decrease of x
(D) K_p remains constant with change in P and x decrease with pressure

4. Maximum value ($n + \ell + m$) for an unpaired electron in second excited state of chlorine $_{17}Cl$ is
(A) 28
(B) 8
(C) 7
(D) none of these

Space for rough work

5. If 40 ml of 0.2 M CH_3COOH is titrated with 0.2 M NaOH. How many ml of base must be added to form a buffer solution with greatest buffer capacity?
(A) 10 ml
(B) 20 ml
(C) 30 ml
(D) 40 ml

6. The molecular shapes of SF_4, SiF_4 and XeF_4 are:
(A) the same, with 2, 0 and 1 lone pair of electrons respectively
(B) the same, with 1, 1 and 1 lone pair of electrons respectively
(C) different with 0, 1 and 2 lone pair of electrons respectively
(D) different with 1, 0 and 2 lone pair of electrons respectively

7. The dipole moment of CH_3OH and CH_3SH are related as
(A) $CH_3OH > CH_3SH$
(B) $CH_3OH < CH_3SH$
(C) $CH_3OH = CH_3SH$
(D) cannot predicted

8. A hypothetical reaction, $A_2 + B_2 \rightarrow 2AB$, follows the mechanism as given below
$A_2 \rightleftharpoons A + A \quad (\text{fast})$
$A + B_2 \rightarrow AB + B(\text{slow})$
$A + B \rightarrow AB(\text{fast})$
the order of overall reaction is
(A) 2
(B) 1
(C) $1\frac{1}{2}$
(D) zero

9. The ratio of the atoms of U^{238} and Pb^{206} in a sample of rock is found to be 4:1. The $t_{1/2}$ of U^{238} is 4.5×10^9 years. How long ago the solidification of the rock could have occurred? (log 2 = 0.30)
(A) 1.35×10^{10} years
(B) 1.5×10^9 years
(C) 2.5×10^8 years
(D) 9.0×10^9 years

10. A system in equilibrium is described by the gaseous phase equation
$\text{Heat} + SO_2Cl_2 \rightleftharpoons SO_2 + Cl_2$
Which of the following statements is true?
(A) addition of Cl_2 will shift the equilibrium towards right & temperature is raised
(B) addition of Cl_2 will shift the equilibrium towards left & temperature is raised
(C) withdrawal of Cl_2 & SO_2 will shift the equilibrium towards right & temperature is raised
(D) withdrawal of SO_2 & Cl_2 will shift the equilibrium towards left & temperature is lowered

Space for rough work

11. The difference n^{th} and $(n + 1)^{th}$ Bohr's radius of H-atoms is equal to $(n-1)^{th}$ Bohr's radius. Hence the value of 'n' is:
(A) 1 (B) 2
(C) 3 (D) 4

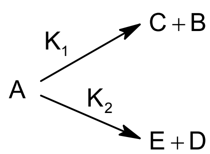
12. The ratio of $(E_2 - E_1)$ to $(E_4 - E_3)$ for the hydrogen atom is approximately equal to
(A) 10 (B) 15
(C) 17 (D) 12

13. The pK_a of weak acid HA is 4.8 and the pK_b of a weak base BOH is 4.78. The pH of an aqueous solution of corresponding salt BA will be:
(A) 9.22 (B) 8.73
(C) 7.01 (D) 4.79

14. The ortho and para-hydrogens possess:
(A) Same physical properties but different chemical properties.
(B) Different physical properties but same chemical properties.
(C) Same chemical and physical properties.
(D) Different physical and chemical properties.

15. Which of the following statement is incorrect?
(A) $LiHCO_3$ does not exist in the solid state.
(B) Potassium superoxide is paramagnetic in nature.
(C) Solubility of $Ba(OH)_2$ in water is more than the solubility of $Mg(OH)_2$.
(D) Li_2CO_3 is more stable than Rb_2CO_3.

16. A substance A (s) undergoes following reaction:



$\left[K_1 = 1\ \text{min}^{-1}, K_2 = 4\ \text{min}^{-1}\right]$

The mole fraction of D in the final mixture will be:
(A) 0.25 (B) 0.2
(C) 0.5 (D) 0.4

17. 0.85 g of a Boron-Hydrogen compound created a pressure of 0.85 atm in a bulb of 480 mL at 97ºC. Analysis showed it to be 86% boron. The molecular formula of compound is
(A) B_2H_6 (B) B_4H_9 (C) B_5H_{10} (D) B_5H_9

18. Arrange the electron represented by the following sets of quantum number in increasing order of energy.

1. $n = 4 \quad \ell = 0 \quad m = 0 \quad ms = +\frac{1}{2}$
2. $n = 3 \quad \ell = 1 \quad m = 1 \quad ms = -\frac{1}{2}$
3. $n = 3 \quad \ell = 2 \quad m = 0 \quad ms = +\frac{1}{2}$
4. $n = 3 \quad \ell = 0 \quad m = 0 \quad ms = -\frac{1}{2}$

(A) 4 < 2 < 1 < 3 (B) 2 < 1 < 3 < 4 (C) 4 < 1 < 2 < 3 (D) 4 < 1 < 3 < 2

19. The incorrect statement among the following is
(A) The first IE of Al is less than the first IE of Mg.
(B) The second IE of Mg is greater than the second IE of Na
(C) The first IE of Na is less than the first IE of Al
(D) The third IE of Mg is greater than the third IE of Al

20. 250 mL of N/10 HCl completely neutralizes a given amount of $CaCO_3$. When all calcium chloride is converted to calcium sulphate then into plaster of paris. The mass of plaster of paris obtained is
(A) 3.6 g (B) 1.7 g (C) 1.81 g (D) 3.4 g

21. Increasing order of bond length of the following species is $O_2^-, N_2, O_2^{2-}, O_2^+, O_2$
(A) $N_2 < O_2^- < O_2^{2-} < O_2 < O_2^+$
(B) $O_2^{2-} < O_2^- < O_2 < O_2^+ < N_2$
(C) $N_2 < O_2 < O_2^+ < O_2^{2-} < O_2^-$
(D) $N_2 < O_2^+ < O_2 < O_2^- < O_2^{2-}$

22. A metal X when dissolved in liquid NH_3, it gives a blue coloured solution which have good reducing property and reduce the bismuth to form a compound Y. X react with O_2 to form Z. Z used in breathing mask to liberate a gas D. X gives lilac/violet colour in Flame. X, Y, Z and D respectively are
(A) K, K_3Bi, K_2O_2, CO_2
(B) K, K_3Bi, KO_2, O_2
(C) K, K_2Bi, K_2O, O_2
(D) Na, Na_2Bi, Na_2O_2, O_2

23. A metal X reacts with water to form Y. Y reacts with chlorine gas to form Z. X reacts with nitrogen to give Q. X gives brick red colour in flame. X, Y, Q and Z respectively is
(A) Ca, $Ca(OH)_2$, Ca_2N_3 and $CaOCl_2$
(B) Ca, $Ca(OH)_2$, Ca_3N_2 and $Ca(OCl)_2$
(C) Sr, $Sr(OH)_2$, Sr_3N_2 and $SrOCl_2$
(D) Ca, $Ca(OH)_2$, $Ca(OCl)_2$ and Ca_3N_2

24. The correct order of the ionic character for PbF_2, $PbCl_2$, $PbBr_2$ and PbI_2 is
(A) $PbF_2 > PbCl_2 > PbBr_2 > PbI_2$
(B) $PbF_2 < PbCl_2 < PbBr_2 > PbI_2$
(C) $PbF_2 < PbCl_2 > PbBr_2 > PbI_2$
(D) $PbF_2 < PbCl_2 < PbBr_2 < PbI_2$

25. If the ionization energy of H-atom is W, then the energy of the electron in its first excited state is equal to
(A) –W (B) -2W (C) –W/2 (D) –W/4

26. In the periodic table, the element with Atomic number 38 belong to
(A) Period four and group two
(B) Period Four and group four
(C) Period three and group four
(D) Period five and group two

27. $BrO_3^- + Br^- \longrightarrow Br_2 + H_2O$
If 50 ml 0.1 M BrO_3^- is mixed with 30 mL of 0.5 M Br^- solution that contains excess of H^+ ions, the moles of Br_2 formed are
(A) 6.0×10^{-4} (B) 1.2×10^{-4} (C) 9.0×10^{-3} (D) 1.8×10^{-3}

28. In the mixture of ($NaHCO_3$ + Na_2CO_3), volume of HCl required is x mL with phenolphthalein indicator and then y mL with methyl orange indicator in same titration. The volume of HCl for complete neutralization of $NaHCO_3$ in the mixture is
(A) y (B) 2x (C) y – 2x (D) x/2

29. The vapour density of a mixture containing NO_2 and $N_2O_4$is 38.3 at 27ºC. The mole of NO_2 in 100 mole mixture is
(A) 30.5 (B) 33.5 (C) 27.8 (D) 43.6

30. Which does not have sp hybridized central atom
(A) XeF_2 (B) C_2H_2 (C) CO_2 (D) BeH_2

space for rough work

(PART – C)
MATHEMATICS

1. Let f(x+T)=f(x), where T>0, The minimum possible value of T for $f(x) = \sin^2x - \cot^2x + \sec^2x.\mathrm{cosec}^2x - \tan^2x + \cos^2x$.
(A)1 (B) $\pi/2$ (C) π (D) None of these.

2. Let f : R $\to$ R and g : R $\to$ R be two one–one and onto functions such that they are the mirror images of each other about the line y = a. If h(x) = f(x) + g(x), then h(x) is
(A) one-one onto (B) one–one into (C) many one-one (D) many one into

3. Find the range of $f(x) = \log_{[x-1]}(\sin x)$ where [.] represent greatest integer function
(A) $(-\infty, 0]$ (B) $(1, \infty)$ (C) (0, 1) (D) none of these

4. $\lim\limits_{x\to\infty} \frac{[2x]}{x}$ (where [x] denotes greatest integer $\le$ x) must be equal to
(A) 0 (B) 1 (C) 2 (D) $\frac{1}{2}$

5. If $f(x) = \lim\limits_{n\to\infty} \frac{2x}{\pi} \cdot \tan^{-1}(nx), x > 0$, then $\lim\limits_{x\to 0^+}[f(x)-1]$ is
{where [.] represents greatest integer function}
(A) 1 (B) –1 (C) 0 (D) none of these

6. Find the Indefinite Integral of $\int \cos 2\theta \ \log\left(\frac{\cot\theta + 1}{\cot\theta - 1}\right) d\theta$
(A) $\frac{1}{2}\sin 2\theta \ln\left(\tan\left(\frac{\pi}{4} + \theta\right)\right) - \frac{1}{2}\ln(\sec 2\theta) + c$
(B) $\frac{1}{2}\sin 2\theta \ln\left(\tan\left(\frac{\pi}{4} + \theta\right)\right) + \frac{1}{2}\ln(\sec 2\theta) + c$
(C) $\frac{1}{2}\sin 2\theta \ln\left(\tan\left(\frac{\pi}{4} - \theta\right)\right) + \frac{1}{2}\ln(\sec 2\theta) + c$
(D) none of these

7. $\frac{x}{a} + \frac{y}{b} = 1$ is a tangent to curve $x = 4t$, $y = \frac{4}{t}$, t is real then
(A) a > 0 , b > 0 (B) a> 0, b < 0 (C) a < 0, b > 0 (D) none of these

8. Let $I_1 = \int_0^3 \frac{\sin x}{\left[\frac{x}{\pi}\right] + \frac{1}{2}} dx$ and $I_2 = \int_{-3}^0 \frac{\sin x}{\left[\frac{x}{\pi}\right] + \frac{1}{2}} dx$, here [.] denotes the greatest integer part. Then
(A) $I_1 = 3I_2$ (B) $I_2 = 3I_1$ (C) $I_1 = I_2$ (D) $I_1 + I_2 = 0$

9. $\int \frac{dx}{x^{1/5}\left(1 + x^{4/5}\right)^{1/2}}$ is
(A) $\sqrt{1 + x^{4/5}} + k$ (B) $\frac{5}{2}\sqrt{1 + x^{4/5}} + k$
(C) $x^{4/5}\left(1 + x^{4/5}\right)^{1/2} + k$ (D) none of these

10. If $\frac{1}{\pi}\int_{-\pi}^{\pi}\frac{x\sin x}{e^x+1}dx = a$, then the number of values of c such that $\int_0^a |c-x|dx = \frac{1}{2}$ is

(A) 0 (B) 1 (C) 2 (D) none of these

11. If $f(x) = \log\left(\frac{1+x}{1-x}\right)$ and $g(x) = \frac{3x+x^3}{1+3x^2}$, then $f(g(x))$ is equal to

(A) f(x) (B) 3f(x)
(C) $(f(x))^3$ (D) f(3x)

12. If $y = \sin^{-1}\sqrt{1-x} + \cos^{-1}\sqrt{x}$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{1}{\sqrt{x(1-x)}}$ (B) $\frac{-1}{\sqrt{x(1-x)}}$

(C) $\frac{1}{\sqrt{x(x-1)}}$ (D) $\frac{-1}{\sqrt{x(x-1)}}$

13. The points on the curve $y = (x^3 - 3x)$ at which the normals are parallel to the line $2x + 18y = 9$ are

(A) (2, 2), $(2, -2)$ (B) $(2, -2)$, $(-2, -2)$
(C) $(2, -2)$, $(-2, 2)$ (D) (2, 2), $(-2, -2)$

14. The function $f(x) = \int_{-2}^{x} t(e^t-1)(t-1)(t-2)^3(t-3)^5\,dt$ has local minimum at x equals to

(A) 0, 4 (B) 1, 3
(C) 0, 2 (D) 2, 4

15. If minimum value of $f(x) = x^2 + 2bx + 2c^2$ is greater than the maximum value of $g(x) = -x^2 - 2cx + b^2$, for real x,, then

(A) $|c| > |b|\sqrt{2}$ (B) $|c|\sqrt{2} > |b|$
(C) $0 < |c| < \sqrt{2}|b|$ (D) $|c|\sqrt{2} < |b|$

16. The value of $\int \frac{(1+\cos 4x)}{(\cot x - \tan x)}dx$ is equal to

(A) $\frac{1}{8}\cos 4x + c$ (B) $-\frac{1}{8}\cos 4x + c$

(C) $\frac{1}{8}\sin 4x + c$ (D) $-\frac{1}{8}\sin 4x + c$

17. If $\int \sec^4 x \,.\, \mathrm{cosec}^2 x\, dx = K\tan^3 x + L\tan x + M\cot x + c$, then

(A) $K = \frac{1}{3}, L = 1, M = 2$ (B) $K = \frac{1}{3}, L = 2, M = -1$

(C) $K = \frac{1}{3}, L = 0, M = 1$ (D) $K = \frac{1}{3}, L = 2, M = 1$

18. The value of $\int e^x\left(\frac{x^2+1}{(x+1)^2}\right)dx$ is equal to

(A) $e^x\left(\frac{x+1}{x-1}\right) + c$ (B) $e^x\left(\frac{x-1}{x+1}\right) + c$

(C) $e^x(x+1) + c$ (D) $\frac{e^x}{(x+1)} + c$

19. The value of $\int \left\{ \frac{(\ln x - 1)}{\left(1+(\ln x)^2\right)} \right\}^2 dx$ is equal to

(A) $\frac{x}{(\ln x)^2+1}+c$ (B) $\frac{x-e^x}{\left(1+x^2\right)}+c$

(C) $\frac{x}{\left(x^2+1\right)}+c$ (D) $\frac{\ln x}{(\ln x)^2+1}+c$

20. The value of $\int_0^{\pi/2} \frac{\sin x \cos x}{\left(1+\sin^4 x\right)} dx$ is equal to

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{6}$ (D) $\frac{\pi}{8}$

21. The value of $\int_0^1 \frac{\left(\sin^2 x+\cos^4 x\right)}{\left(\cos^2 x+\sin^4 x\right)} \,.\, x^5 dx$ is equal to

(A) $\frac{1}{3}$ (B) $\frac{1}{4}$

(C) $\frac{1}{6}$ (D) $\frac{1}{5}$

22. If $S_n = \left[\frac{1}{1+\sqrt{x}}+\frac{1}{2+\sqrt{2x}}+........+\frac{1}{n+\sqrt{n^2}}\right]$, then $\lim\limits_{n\to\infty}(S_n)$ is equal to

(A) ln2 (B) ln4
(C) ln16 (D) ln8

23. Let f be a real valued invertible function such that $f\left(\frac{3x-5}{x+2}\right)=100x+15,\ x \neq 2$. Then find the value of $f^{-1}(2015)$.

(A) 2016 (B) $\frac{6034}{2015}$ (C) $\frac{5}{2}$ (D) $\frac{2}{5}$

24. $\lim\limits_{n\to\infty}\left[\frac{1}{n^2+1}+\frac{2}{n^2+2}+\frac{3}{n^2+3}+....+\frac{1}{n+1}\right]=$

(A) 0 (B) 1/2 (C) 1 (D) 2

25. $f(x)=\max\{\tan x, \cot x\}$ is not differentiable at

(A) $\left\{x \mid x=\frac{n\pi}{4}, n \in I\right\}$ (B) $\left\{x \mid x=\frac{n\pi}{3}, n \in I\right\}$ (C) $\left\{x \mid x=\frac{n\pi}{8}, n \in I\right\}$ (D) None of these

26. If $y=e^{\sqrt{x}}+e^{-\sqrt{x}}$ then $xy_2+(1/2)y_1$ is equal to

(A) y (B) $x\left(e^{\sqrt{x}}+e^{-\sqrt{x}}\right)$ (C) $(1/4)y$ (D) $\sqrt{x}y$

27. Let f be a function satisfying $f(xy)=\frac{f(x)}{y}$ for all positive real numbers x and y if $f(30)=20$, then the value of $f(40)$ is

(A) 40 (B) 20 (C) 15 (D) 60

28. Let $f(x)=\lim\limits_{n\to\infty}\dfrac{\log(2+x)-x^{2n}\sin x}{1+x^{2n}}$ Then

(A) f is continuous at $x=1$

(B) $\lim\limits_{x\to1+}f(x)\neq\lim\limits_{x\to1-}f(x)$

(C) $\lim\limits_{x\to1+}f(x)=\sin 1$

(D) $\lim\limits_{x\to1-}f(x)$ doesn't exist

29. Let $f(x)=\cot^{-1}(x^2-4x+5)$, then range of $f(x)$ is equal to

(A) $\left(0,\dfrac{\pi}{2}\right)$

(B) $\left(0,\dfrac{\pi}{4}\right]$

(C) $\left[0,\dfrac{\pi}{4}\right)$

(D) none of these

30. If $0<a<b<\dfrac{\pi}{2}$ and $f(a,b)=\dfrac{\tan b-\tan a}{b-a}$, then

(A) $f(a,b)\ge 2$

(B) $f(a,b)>1$

(C) $f(a,b)\le 1$

(D) none of these

Space for rough work

FIITJEE – JEE (Mains)

PHYSICS, CHEMISTRY & MATHEMATICS

Applicable for Class – XII PASS

PHASE - 1

ANSWER KEY

PHYSICS

1.	B	2.	C	3.	C	4.	C	5.	B	6.	A	7.	C
8.	D	9.	A	10.	B	11.	A	12.	C	13.	A	14.	C
15.	C	16.	A	17.	B	18.	C	19.	C	20.	D	21.	A
22.	C	23.	C	24.	C	25.	B	26.	A	27.	C	28.	B
29.	A	30.	C										

CHEMISTRY

1.	A	2.	A	3.	D	4.	C	5.	B	6.	D	7.	A
8.	C	9.	B	10.	C	11.	D	12.	B	13.	C	14.	B
15.	D	16.	D	17.	D	18.	A	19.	B	20.	C	21.	D
22.	B	23.	B	24.	A	25.	D	26.	D	27.	C	28.	C
29.	B	30.	A										

MATHS

1.	B	2.	D	3	A	4.	C	5.	B	6.	A	7.	A
8.	C	9.	B	10.	C	11.	B	12.	B	13.	D	14.	B
15.	A	16.	B	17.	B	18.	B	19.	A	20.	D	21.	C
22.	B	23.	C	24.	B	25.	A	26.	C	27.	C	28.	B
29.	B	30.	B										

SOLUTIONS

PHYSICS

1. $W_{agent} = \Delta K + \Delta U$
for minimum work done $\Delta K = 0$
$W_{agent} = \Delta U = mgR$

2. The total time from A to C

$$t_{AC} = t_{AB} + t_{BC} = \frac{T}{4} + t_{BC}$$

Where T = Time period of oscillation of spring-mass system t_{BC} can be given by

$$BC = AB \sin\left(\frac{2\pi}{T}\right)t_{BC}$$

Putting $\frac{BC}{AB} = \frac{1}{2}$, we get

$$t_{BC} = \frac{T}{12}$$

$$\therefore \quad t_{BC} = \frac{2\pi}{3}\sqrt{\frac{m}{k}}$$

3. $E = \frac{1}{2}mv^2 \Rightarrow E = \frac{1}{2}v^2$ {Since m = 1}
By equation of Trajectory

$$y = x\tan\theta - \frac{g}{2}\frac{x^2}{u^2\cos^2\theta},$$

$$0 = R\tan\theta - \frac{g}{2}\frac{R^2}{v^2\cos^2\theta}$$

$$R\tan\theta = \frac{g}{2}\frac{R^2}{v^2\cos^2\theta} \;;\; \sin\theta = \frac{g}{2}\frac{R}{v^2\cos\theta}$$

$$\sin 2\theta = \frac{gR}{v^2}$$

$$\sin 2\theta = \left[\frac{gR}{2E}\right] \;;\; \theta = 0.5\sin^{-1}\left[\frac{gR}{2E}\right]$$

4. On Rough surface
At any distance from the end

$$T_x = F_0\left[1 - \frac{x}{L}\right]$$

Hence $T_x = F_0\left[1 - \frac{x}{L}\right] = 20 \times \frac{1}{2} = 10\,\text{Newton}$

$F_{shaded} = M_{shaded}\,A = \frac{M}{3}A$

$A = \frac{F}{M}$

$F_{shaded} = \frac{F}{3}$.

6. $0 = u\cos 30° - g\sin 30°\, t$

$$t = \frac{u\cos 30°}{g\sin 30°} \qquad .....(A)$$

$$-H\cos 30° = -u\sin 30° t - \frac{1}{2} g\cos 30° t^2$$

By equation (A) and (B), we get

$$H = \frac{u^2}{g}\left[1+\frac{\cot^2\alpha}{2}\right] \; v = \sqrt{\frac{2gH}{5}} \quad \{\alpha = 30°\}$$

7. (C) $y = r\sin\theta$

$$\frac{dy}{dt} = \frac{dr}{dt}\sin\theta + \cos\theta\frac{d\theta}{dt}.r = 3.6\,m/s$$

8. D P110603

$$mg\frac{L}{2}\sin 60° = Mg\frac{L}{2}\sin 30°$$

9. A P110610
Conceptual

10. B P110604
Net external force on the system is zero.

11. A P110604
Velocity of CM does not change as there is no external force.

12. C P110009
Maximum velocity of left side will be with C and then in decreasing order for rest of the balls.

13. A P110602

$$X_{cm} = \frac{\int dm}{\int dm} = \frac{\int x\lambda dx}{\int \lambda dx}$$

14. C P110607

$v\cos\phi = eu\cos\theta$

$v\sin\phi = u\sin\theta$

15. C P110602
Conceptual

16. A

After first impact $(0)^2 - \left(e\sqrt{2gh}\right)^2 = 2(-g)h_1$

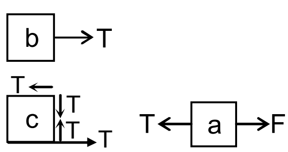
$\Rightarrow \; h_1 = e^2h \qquad \Rightarrow \; h_2 = e^2h_1 = e^4h \Rightarrow h_n = e^{2n}h$

17. B

By mass removal method $\left(\frac{4}{3}\pi(R)^3 - \frac{4}{3}\pi\left(\frac{R}{2}\right)^3\right)x = \left(\frac{4}{3}\pi\left(\frac{R}{2}\right)^3\right)\times x\frac{R}{2}$

$x = \frac{R}{14}$ cm.

18. C



$F - T = 10\times a$

$T = 10\times a$

$F = 20a \Rightarrow a = 1$

$T = 10N, \quad a_a = a_b = 1, \; a_c = 0$

19. C

Maximum velocity $v_{max} = \sqrt{2as}$

$v_{avg} = \frac{6s}{T} \qquad T = t_1 + t_2 + t_3$

$t_1 = \sqrt{\frac{2s}{a}}, \qquad t_2 = \frac{2s}{v_{max}} = \sqrt{\frac{2s}{a}}$

Retardation $\Rightarrow$ B

$\beta = \frac{v_{max}^2}{6s}; \qquad \beta = \frac{2a \times s}{6 \times s} = \frac{a}{3}$

$t_3 = \frac{v_{max}}{\beta} = 3\sqrt{\frac{2a \times s}{a}} = 3\sqrt{\frac{2s}{a}}; \qquad T = 5\sqrt{\frac{2s}{a}}$

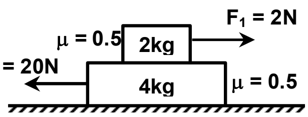
$v_{avg} = \frac{6}{5}\frac{5}{\sqrt{\frac{2s}{a}}}; \qquad v_{avg} = \frac{6}{5}\sqrt{\frac{as}{2}}$

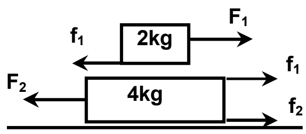
$\frac{v_{avg}}{v_{max}} = \frac{6}{5}\sqrt{\frac{as}{2}} \times \frac{1}{\sqrt{2as}} = \frac{3}{5}.$

20. D

$a = \frac{F - m_1 g}{m_1}$

21. A





22. C

From work energy theorem $W_{mg} + W_p = \frac{1}{2}m\left(v_1^2 - v^2\right)$

$-2Fh = \frac{1}{2}m\left(v_f^2 - v^2\right)$ where $h = \frac{v^2}{2\left(g + \frac{F}{m}\right)} \Rightarrow v_f = v\sqrt{\frac{mg - F}{mg + F}}$

23. C

24. C

$v = -\frac{1}{2}s + 50 \Rightarrow \frac{ds}{dt} = \frac{-s}{2} + 50$

$\int_{x=0}^{s=100} \frac{ds}{-\frac{s}{2} + 50} = \int_{t=0}^{t} dt \Rightarrow \frac{1}{-1/2}\left(\ell n\left(\frac{-s}{2} + 50\right)\right)_0^{100} = (t) \Rightarrow -2\left(\ell n - \ell n50\right) = t \Rightarrow t = \infty$

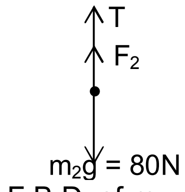
25. B

26. A

$e = \frac{p_2^f - p_1^f}{p_1^i - p_2^i} = \frac{J - (P - J)}{P}$

27. C

As the springs are fixed to the horizontal and have the same natural length, hence if one spring is compressed, the other must be expanded. Hence, the compression will be negative.



F.B.D of m_2

F.B.D of m_2 $T + F_2 = 80$ N

and $F_2 = 70 \times 0.5 = 35$ N

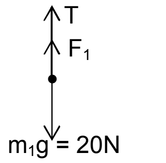
$\therefore$ $T = 80 - 35 = 45$ N

F.B.D of m_1 $T + F_1 + mg$

or $45 = F_1 + 20$

or $F_1 = 25$ N

$\therefore$ $x_1 = \frac{25}{k_1} = \frac{25}{50} = 0.5$m



F.B.D of m_1

$\therefore$ Compression in first spring = –0.5 m

28. B

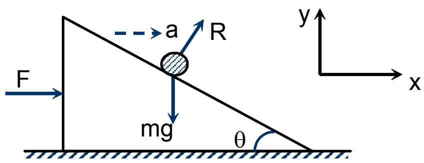
29. A

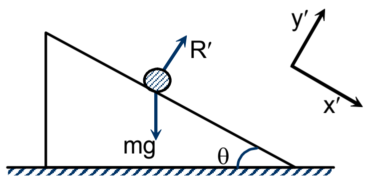
The ball moves along x-axis with an acceleration, say a, whereas it is in equilibrium along y axis

$\Rightarrow$ $\Sigma F_x = 0 \Rightarrow R\sin\theta = ma$

and $\Sigma F_y = 0 \Rightarrow R\cos\theta = mg$

$\Rightarrow$ $R = mg\,Sec\theta$ ...(1)





When F = 0, Next, the ball moves down the plane, along X′ axis, say with an acceleration a′

$\Rightarrow$ $mg\sin\theta = ma'$

The ball does not move perpendicular to the plane. It is in equilibrium along y′- axis

$\Rightarrow$ $mg\cos\theta - R' = 0$

$\Rightarrow$ $R' = mg\cos\theta$ ...(2)

Dividing (1) by (2) $\Rightarrow$ $\frac{R}{R'} = \sec^2\theta$

30. C

Hints & Solutions

PART -B: CHEMISTRY

5. At greatest buffer capacity [salt] = [undissociated acid].

11. $r = \frac{0.529 \times n^2}{1} A^{o}$

$\therefore$ $0.529\,[(n + 1)^2 - n^2] = 0.529\,(n - 1)^2$

on solving, n = 4

13. $pH = \frac{1}{2}[pK_W + pK_a - pK_b] = \frac{1}{2}[14 + 4.8 - 4.78] = 7.01$

15. Li_2CO_3 is less stable than Rb_2CO_3.

16. In final mixture [B] = [C] and [E] = [D] and $\frac{[B]}{[D]} = \frac{K_1}{K_2}$, $X_D = \frac{K_2}{2K_1 + 2K_2} = \frac{4}{10} = 0.4$

17. $PV = \frac{w}{M}RT$

$M = \frac{wRT}{PV}$

Number of Boron atom = $\frac{54.4}{10.8} = 5.03 \simeq 5$

Mass of H = 63.28 – 54.4 = 8.88

$\therefore$ Number of H = 9

$\therefore$ Molecular formula = B_5H_9

18. $E \propto (n + \ell)$

If $n + \ell$ are same, orbital having greater value of n possess higher energy

$\therefore 4 < 2 < 1 < 3$

20. Meq of $CaCO_3$ = Meq of $CaCl_2$ = Meq of $CaSO_4$ = Meq of HCl

Let W be mass of plaster of paris ($CaSO_4 \frac{1}{2}H_2O$)

$\frac{W}{M/2} = 25 \times 10^{-3}$

W = 145 × 25 × 10^{-3}

W = 1.81 g

21. Bond length $\propto \frac{1}{\text{bond order}}$

Order of bond length

$N_2 < O_2^+ < O_2 < O_2^- < O_2^{2-}$

22. K gives violet/Lilac colour in flame

$Bi \xrightarrow{K/NH_3(Liq)} \underset{Y}{K_3Bi}$

$\underset{(X)}{K} + O_2 \longrightarrow \underset{Z}{KO_2}$

$4KO_2 + 2CO_2 \longrightarrow 2K_2CO_3 + O_2$

24. $PbF_2 > PbCl_2 > PbBr_2 > PbI_2$

25. IE of H atom = W = $E_\infty - E_1$

$W = 2.18 \times 10^{-18}$ J

First excited state n = 2

$E_2 = -\frac{W}{4}$

26. Z = 38

$1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^{10}\ 4s^2\ 4p^6\ 5s^2$

Group = 2

Period = 5

27. $6H^+ + \underset{5\text{Milimole}}{BrO_3^-} + \underset{15\text{milimole}}{5Br^-} \longrightarrow 3Br_2 + 3H_2O$

Br^- is limiting reagent

15 milimole of Br^- gives $\frac{3}{5} \times 15 = 9$ milimole

29. $2NO_2 \rightleftharpoons N_2O_4$

Molecular weight of mixture of NO_2 and N_2O_4 = 38.3 × 2 = 76.6

x moles of NO_2 are present in mixture

$x \times 46 + (100 - x)92 = 33.5\,\text{mol}$

30. A

SOLUTIONS - MATHS

1. B

$f(x)=3,\ \forall\ x \neq \frac{n\pi}{2}, n \in Z$.

2. D

If $f(x) = a + \lambda$, at some x, then $g(x) = a - \lambda$, at that x

So, $h(x) = 2a$, a constant function.

3. A

$1 \geq \sin x > 0 \Rightarrow x \in (0, \pi)\ \forall\ x \in [0, 2\pi]$

domain $x \geq 3$ and $x \in (2n\pi, 2n\pi + \pi)$ where $n \in$ non negative integer.

$\log_{[x-1]} \sin x = \frac{\log(\sin x)}{\log[x-1]}$ hence range R = $(-\infty, 0]$

4. C

$\lim_{x\to\infty} \frac{[2x]}{x} = \lim_{x\to\infty} \frac{2x - \{2x\}}{x}$

({y} denotes fractional part of y)

$= \lim_{x\to\infty} \left[2 - \frac{\text{finite}}{x}\right] = 2$.

5. B

$f(x) = \lim_{n\to\infty} \frac{2x}{\pi} \tan^{-1}(nx) = x, x > 0$

$\therefore \lim_{x\to 0^+} [f(x) - 1] = \lim_{x\to 0^+} [x - 1] = -1$

6. A

I = $\int \cos 2\theta \ \log\left(\frac{1+\tan\theta}{1-\tan\theta}\right) d\theta = \int \cos 2\theta \ \log \ \tan\left(\frac{\pi}{4} + \theta\right) d\theta$

We will integrate it by parts,

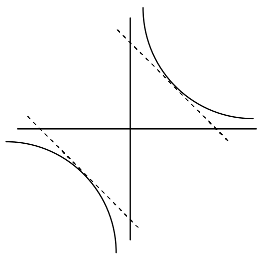
$I = \frac{1}{2} \sin 2\theta \log\tan\left(\frac{\pi}{4} + \theta\right) - \frac{1}{2}\int \sin 2\theta \cdot 2\sec 2\theta d\theta + c1$

[Using $\frac{d}{dx} \log\tan\left(\frac{\pi}{4} + \theta\right) = 2\sec 2\theta$]

$= \frac{1}{2}\sin 2\theta \log\tan\left(\frac{\pi}{4} + \theta\right) - \int \tan 2\theta d\theta + c_1$

$= \frac{1}{2}\sin 2\theta \log\tan\left(\frac{\pi}{4} + \theta\right) - \frac{1}{2}\log\sec 2\theta + c$

7. A



$x = 4t, \quad y = \frac{4}{t}$

$\Rightarrow \quad xy = 16$ ….(1)

$\frac{x}{a} + \frac{y}{b} = 1$ can only be tangent to (1)

if a, b both have same sign i. e. either a, > 0, b > 0 or a < 0, b < 0

8. C

$I_1 = \int_0^3 2\sin x dx$

$I_2 = \int_{-3}^{0} \frac{\sin x}{\left(-1 + \frac{1}{2}\right)} = -\int_{-3}^{0} 2\sin x\, dx = \int_{3}^{0} 2\sin(-t)dt$

$\int_0^3 2\sin x dx = I_1$

9. B

10. C

$I = \frac{1}{\pi}\int_{-\pi}^{\pi}\frac{x\sin x}{e^{-x}+1}dx$

$\Rightarrow 2I = \frac{1}{\pi}\int_{-\pi}^{\pi} x\sin x\, dx$

$\Rightarrow I = \frac{1}{\pi}\int_0^{\pi} x\sin x\, dx = 1$

Also $\int_0^1 |c-x|dx = \frac{1}{2} \Rightarrow c = 0,1$

11. **B**

$f(g(x)) = \log\left\{\frac{1+\left(\frac{3x+x^3}{1+3x^2}\right)}{1-\left(\frac{3x+x^3}{1+3x^2}\right)}\right\} = \log\left(\frac{1+x}{1-x}\right)^3 = 3f(x)$

12. **B**

$\frac{dy}{dx} = \frac{1}{\sqrt{1-(1-x)}} \cdot \frac{1}{2\sqrt{1-x}}\times(-1) + \frac{(-1)}{\sqrt{1-x}}\times\frac{1}{2\sqrt{x}}$

$= \frac{-1}{2\sqrt{x}\sqrt{1-x}} - \frac{1}{2\sqrt{x}\sqrt{1-x}}$

$= \frac{-1}{\sqrt{x(1-x)}}$

13. **D**

$\frac{dy}{dx} = 3x^2 - 3 = 3(x^2-1)$

Slope of the line $= \frac{-2}{18} = \frac{-1}{9}$

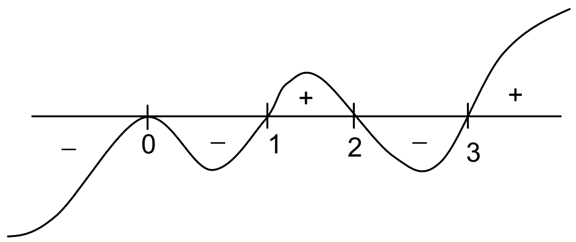
Since normal is parallel to the given line

$\therefore\ 3(x^2-1) = 9 \ \Rightarrow x^2 - 1 = 3 \ \Rightarrow x^2 = 4 \ \Rightarrow x = \pm 2$

$\therefore\ y = (8-6),\ (-8+6) = 2,\ -2$

$\therefore$ (2, 2) & $(-2,\ -2)$ are required points

14. **B**



$f'(x) = x(e^x-1)(x-1)(x-2)^3(x-3)^5$

Clearly, x = 1, 3 are the points of local minima.

15. **A**

$$f(x) = (x+b)^2 + (2c^2 - b^2) \geq 2c^2 - b^2$$

$$g(x) = (b^2 + c^2) - (x+c)^2 \leq b^2 + c^2$$

$$\therefore\ (2c^2 - b^2) > (b^2 + c^2)$$

$$\Rightarrow\ c^2 > 2b^2 \ \Rightarrow |c| > \sqrt{2}|b|$$

16. **B**

$$I = \int \frac{2\cos^2 2x}{(\cos^2 x - \sin^2 x)} \ .\ \sin x \cos x \ dx$$

$$= \int \cos 2x \ .\ \sin 2x \ dx = \frac{1}{2}\int \sin 4x \ dx$$

$$= -\frac{1}{8}\cos 4x + c$$

17. **B**

$$I = \int (1+\tan^2 x)\ .\ \left(1+\frac{1}{\tan^2 x}\right)\sec^2 x \ dx$$

Put $\tan x = t \ \Rightarrow \sec^2 x \ dx = dt$

$$\therefore\ I = \int (1+t^2)\left(1+\frac{1}{t^2}\right)dt$$

$$= \int \frac{(t^4 + 2t^2 + 1)}{t^2}dt = \int\left(t^2 + 2 + \frac{1}{t^2}\right)dt$$

$$= \frac{t^3}{3} + 2t - \frac{1}{t} + c$$

$$= \frac{1}{3}\tan^3 x + 2\tan x - \cot x + c$$

$$\therefore\ K = \frac{1}{3},\ L = 2,\ M = -1$$

18. **B**

$$I = \int e^x \left\{\frac{(x+1)^2 - 2x}{(x+1)^2}\right\}dx$$

$$= \int e^x \left\{1 - \frac{2x}{(x+1)^2}\right\}dx$$

$$= e^x - 2\int e^x \left\{\frac{(x+1)-1}{(x+1)^2}\right\}dx$$

$$= e^x - 2\int e^x \left\{\frac{1}{(x+1)} - \frac{1}{(x+1)^2}\right\}dx$$

$$= e^x - 2\frac{e^x}{(x+1)} + c$$

$$= e^x\left[\frac{x+1-2}{x+1}\right] + c$$

$$= e^x\left(\frac{x-1}{x+1}\right) + c$$

19. **A**

$$I = \int \frac{(\ln x - 1)^2}{\left(1+(\ln x)^2\right)^2}dx, \quad \text{Put } \ln x = t$$

$$\Rightarrow x = e^t$$

$$= \int \frac{(t-1)^2}{(t^2+1)^2} \cdot e^t dt \qquad \Rightarrow dx = e^t dt$$

$$= \int \left\{ \frac{1}{1+t^2} - \frac{2t}{(1+t^2)^2} \right\} e^t dt$$

$$= \frac{e^t}{1+t^2} + c = \frac{x}{1+(\ln x)^2} + c$$

20. **D**

Put $\sin^2 x = t \quad \Rightarrow 2\sin x \cos x dx = dt$

$$\therefore \ I = \frac{1}{2}\int_0^1 \frac{dt}{1+t^2} = \frac{1}{2}\left[\tan^{-1} t\right]_0^1 = \frac{1}{2}\left[\frac{\pi}{4} - 0\right] = \frac{\pi}{8}$$

21. **C**

$$I = \int_0^1 \frac{(1-\cos^2 x + \cos^4 x)}{\cos^2 x + (1-\cos^2 x)^2} \cdot x^5 dx$$

$$= \int_0^1 \left(\frac{1-\cos^2 x + \cos^4 x}{1+\cos^4 x - \cos^2 x} \right) \cdot x^5 dx = \int_0^1 x^5 dx = \left[\frac{x^6}{6}\right]_0^1 = \frac{1}{6}$$

22. **B**

$$\lim_{n\to\infty}(S_n) = \lim_{n\to\infty} \frac{1}{n} \left[\frac{1}{\frac{1}{n} + \sqrt{\frac{1}{n}}} + \frac{1}{\frac{2}{n} + \sqrt{\frac{2}{n}}} + \dots\dots + \frac{1}{\frac{n}{n} + \sqrt{\frac{n}{n}}} \right]$$

$$= \lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{\left(\frac{r}{n} + \sqrt{\frac{r}{n}}\right)} = \int_0^1 \frac{1}{(x+\sqrt{x})} dx$$

$$= \int_0^1 \frac{1}{\sqrt{x}(\sqrt{x}+1)} dx, \quad \text{Put} \sqrt{x} = t$$

$$\Rightarrow \frac{1}{2\sqrt{x}} dx = dt$$

$$= 2\int_0^1 \frac{dt}{(1+t)} = 2\left[\ln(1+t)\right]_0^1$$

$$= 2[\ln 2 - 0] = \ln 4$$