FIITJEE Phase Test (JEE-Advanced)

BATCHES – 1920

PHYSICS, CHEMISTRY & MATHEMATICS

SET - A

CPT - 1 | **CODE:** | **PAPER - 1**

Time Allotted: 3 Hours **Maximum Marks: 183**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.
- You are not allowed to leave the Examination Hall before the end of the test.

INSTRUCTIONS

Caution: Question Paper CODE as given above MUST be correctly marked in the answer OMR sheet before attempting the paper. Wrong CODE or no CODE will give wrong results.

A. General Instructions

1. Attempt ALL the questions. Answers have to be marked on the OMR sheets.
2. This question paper contains **Three Sections**.
3. **Section-I** is Physics, **Section-II** is Chemistry and **Section-III** is Mathematics.
4. Each **Section** is further divided into **Two Parts**: **Part-A & C** in the OMR. Part-B of OMR to be left unused.
5. Rough spaces are provided for rough work inside the question paper. No additional sheets will be provided for rough work.
6. Blank Papers, clip boards, log tables, slide rule, calculator, cellular phones, pagers and electronic devices, in any form, are not allowed.

B. Filling of OMR Sheet

1. Ensure matching of OMR sheet with the Question paper before you start marking your answers on OMR sheet.
2. On the OMR sheet, darken the appropriate bubble with HB pencil for each character of your Enrolment No. and write in ink your Name, Test Centre and other details at the designated places.
3. OMR sheet contains alphabets, numerals & special characters for marking answers.

C. Marking Scheme For All Two Parts.

(i) **Part-A (01-07) –** Contains seven (07) multiple choice questions which have **One or More** correct answer.
Full Marks: **+4** If only the bubble(s) corresponding to all the correct options(s) is (are) darkened.
Partial Marks: **+1** For darkening a bubble corresponding **to each correct option,** provided NO incorrect option is darkened.
***Zero Marks*: 0** If none of the bubbles is darkened. ***Negative Marks*: –2** In all other cases.
For example, if **(A), (C) and (D)** are all the correct options for a question, darkening all these three will result in **+4 marks**; darkening only **(A) and (D)** will result in **+2 marks**; and darkening **(A) and (B)** will result in **–1** marks, as a wrong option is also darkened

(i) **Part-A (08-13) –** Contains six (06) multiple choice questions which have ONLY ONE CORRECT answer Each question carries **+3 marks** for correct answer and **-1 marks** for wrong answer.

(ii) **Part-C (01-05)** contains five (05) Numerical based questions with single digit integer as answer, ranging from 0 to 9 (both inclusive) and each question carries **+3 marks** for correct answer. **There is no negative marking.**

Name of the Candidate :_____

Batch :_____ **Date of Examination :**_____

Enrolment Number :_____

Useful Data

PHYSICS

Acceleration due to gravity	$g = 10\ m/s^2$
Planck constant	$h = 6.6 \times 10^{-34}$ J-s
Charge of electron	$e = 1.6 \times 10^{-19}$C
Mass of electron	$m_e = 9.1 \times 10^{-31}$ kg
Permittivity of free space	$\varepsilon_0 = 8.85 \times 10^{-12}\ C^2/N\text{-}m^2$
Density of water	$\rho_{water} = 10^3\ kg/m^3$
Atmospheric pressure	$P_a = 10^5\ N/m^2$
Gas constant	$R = 8.314\ J\ K^{-1}\ mol^{-1}$

CHEMISTRY

Gas Constant	R	=	$8.314\ J\ K^{-1}\ mol^{-1}$
		=	0.0821 Lit atm $K^{-1}\ mol^{-1}$
		=	$1.987 \approx 2$ Cal $K^{-1}\ mol^{-1}$
Avogadro's Number	N_a	=	6.023×10^{23}
Planck's Constant	h	=	6.626×10^{-34} Js
		=	6.25×10^{-27} erg.s
1 Faraday		=	96500 Coulomb
1 calorie		=	4.2 Joule
1 amu		=	1.66×10^{-27} kg
1 eV		=	1.6×10^{-19} J

Atomic No.: H=1, He=2, Li=3, Be=4, B=5, C=6, N=7, O=8, F=9, Na=11, Mg=12, Al = 13,
Si = 14, P = 15, S = 16, Cl = 17, Ar =18, K=19, Ca=20,Cr=24, Mn=25, Fe=26, Co=27, Ni=28, Cu=29, Zn=30, As=33, Br = 35, Ag = 47, Si = 21, Sn = 50, Ti = 22,I = 53, Xe = 54, Ba = 56, Pb = 82, U = 92, V = 50.

Atomic masses: H =1, He=4, Li=7, Be=9, B=11, C=12, N=14, O=16, F=19, Na=23, Mg=24, Al=27, Si=28, P=31, S=32, Cl=35.5, K=39, Ca=40, Cr=52, Mn=55, Fe=56, Co=59, Ni=58.7, Cu=63.5, Zn = 65.4, As = 75, Br = 80, Ag = 108, Sn = 118.7, I = 127, Xe = 131, Ba = 137, Pb = 207, U = 238.

SECTION – I : PHYSICS

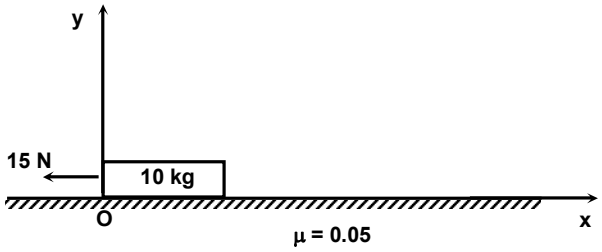
PART – A : (One or more than one Options Correct Type)

This section contains **7 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D), out of which **ONE** or **MORE THAN ONE is correct.**

1. A particle moves in space such that its position vector $\vec{r}(t)$ varies according to $\vec{r}(t) = (\cos t)\hat{i} + (\sin t)\hat{j} + (3t^2 + 1)\hat{k}$, then particle

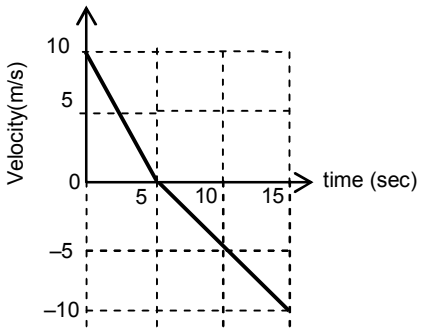
   (A) is moving with constant acceleration
   (B) is moving with continuously increasing speed
   (C) has velocity and acceleration perpendicular to each other at t = 0
   (D) follows path having radius of curvature $\frac{1}{\sqrt{37}}$ m at t = 0

2. A body of mass 10 kg is kept at horizontal rough surface as shown in the figure. The coefficient of friction between body and the surface is 0.05. At t = 0, body is given velocity 10 m/s along positive x-axis, simultaneously, a force of 15 N starts acting along negative x-axis continuously through the motion of Body. Choose the correct graph(s).

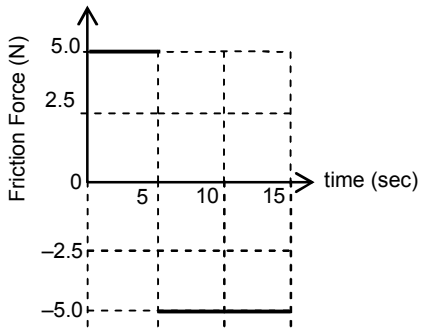


(Physical quantities along positive x-axis, are considered as positive)

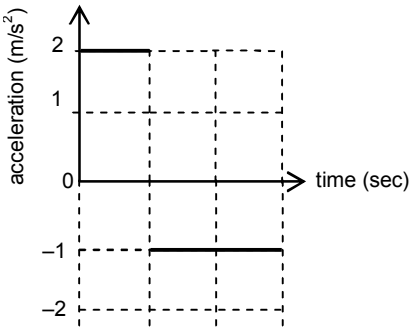
(A)



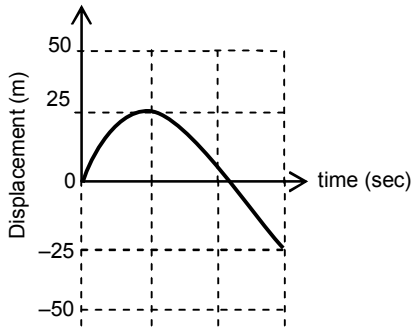
(B)



(C)

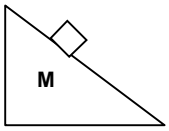


(D)



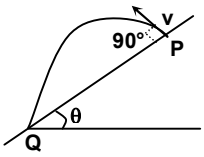
Space for Rough Work

3. A block is kept at the top of a smooth wedge, which in turn is kept on a smooth horizontal surface. Then after releasing the system.



(A) Horizontally the centre of mass will not shift.
(B) Centre of mass moves vertically.
(C) Centre of mass shifts in both directions horizontally as well as vertically.
(D) None of these.

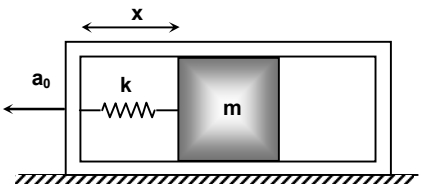
4. If time taken by the projectile to reach Q is T, than PQ =



(A) Tv sin θ (B) Tv cos θ
(C) Tv sec θ (D) Tv tan θ

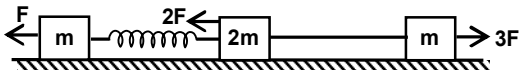
5. A block is suspended by an ideal spring constant K. If the block is pulled down by constant force F and if maximum displacement of block from it's initial position of rest is z, then
(A) z = F/K
(B) z = 2F/K
(C) work done by force F is equal to 2Fz.
(D) increase in potential energy of the spring is $\frac{1}{2}Kz^2$

6. A block of mass m is attached to the frame by a light spring of stiffness k. The frame and block are initially at rest with $x = x_0$, the uncompressed length of the spring. If the frame is given a constant horizontal acceleration a_0 towards left, determine the maximum velocity $(v_{rel})_{max}$ of the block relative to the frame (block is free to move inside frame). Ignore any friction.



(A) $a_0\sqrt{\frac{m}{2k}}$ (B) $a_0\sqrt{\frac{2m}{k}}$
(C) $a_0\sqrt{\frac{m}{k}}$ (D) $\frac{1}{2}a_0\sqrt{\frac{m}{k}}$

7. Three point masses m, 2m and m, connected with ideal spring (of spring constant k) and ideal string as shown in the figure, are placed on a smooth horizontal surface



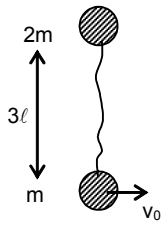
At t = 0, three constant forces F, 2F and 3F start acting on the point masses m, 2m and m respectively, as shown in figure. Find the maximum extension in the spring.

(A) $\frac{2F}{k}$ (B) $\frac{5F}{2k}$
(C) $\frac{3F}{k}$ (D) $\frac{8F}{3k}$

Single Correct Answer Type

This section contains **6 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE is correct.**

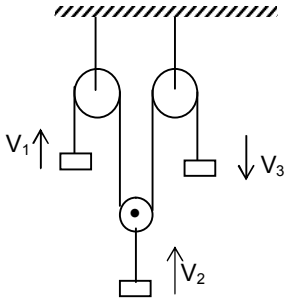
8. Two balls of mass 2m and m are connected by ideal string of length 5ℓ. The system is kept on the horizontal smooth surface. Initial velocity of m is v_0 as shown. The impulse of tension imparted by string to 2m will be



(A) $\frac{12mv_0}{15}$ (B) $\frac{16mv_0}{15}$

(C) $\frac{8mv_0}{15}$ (D) $\frac{4mv_0}{15}$

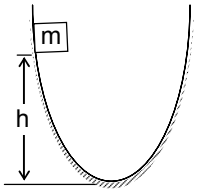
9. A pulley block system is shown in the figure. The instantaneous velocities of the blocks v_1, v_2 and v_3 are shown. The relation between them is



(A) $v_1 = v_3 - 2v_2$ (B) $v_1 = v_3 + 2v_2$

(C) $v_3 = v_1 + v_2$ (D) $v_1 + v_3 = 2v_2$

10. A block of mass m slides down on a smooth parabolic surface from height h as shown. Find its speed at lowermost position



(A) $\sqrt{gh}$ (B) $\sqrt{2gh}$

(C) $\sqrt{5gh}$ (D) data insufficient

11. A block of mass 2 kg was moving along a straight line on a smooth surface with a speed of 5 m/s. At t = 0, a force given by F = (3 + 2t) N directed in the direction of motion of the body starts acting on the block. The kinetic energy of the block after 2 sec is

(A) 20 J (B) 200 J

(C) 100 J (D) none of these.

12. A person wants to drive on the vertical surface of a large cylindrical wooden 'well' commonly known as 'deathwell' in a circus. The radius of the 'well' is 2 meter, and the coefficient of friction between the tyres of the motorcycle and the wall of the well is 0.2. Minimum speed the motorcyclist must have in order to prevent slipping should be (take g = 10 m/s^2)

(A) 10 m/s (B) 15 m/s

(C) 20 m/s (D) 25 m/s

13. There are three vectors $\vec{P}, \vec{Q}$ and $\vec{R}$. The angle between $\vec{P}$ and $\vec{Q}$ is 60º and $\vec{R}$ is perpendicular to the plane containing the vectors $\vec{P}$ and $\vec{Q}$. Which of the following relation is possibly correct ?

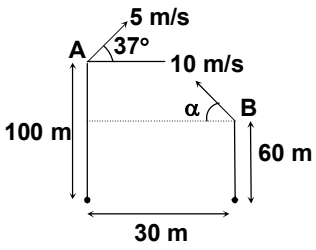
(A) $\vec{P}+\vec{Q}+\vec{R}=0$ (B) $\vec{P}\times\vec{Q}=\vec{R}$

(C) $\vec{P}\times\vec{R}=\vec{Q}$ (D) All of these

PART-C
Integer Answer Type

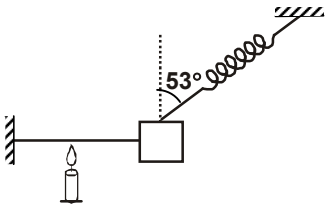
This section contains **5 questions**. The answer to each of the questions is a single digit integer, ranging from **0 to 9**.

1. Two particle are projected simultaneously at point A and point B from two towers as shown in the figure. If they collide then the value of α is (a $\times$ 10 + b) in degree. Find the value of (a $-$ b). (Neglect any type of frictional force acting on the particles during motion. (take g = 10 m/s^2)(where a and b are positive integer, a < 10 and b < 10)

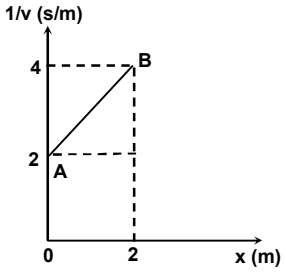


2. Two blocks of masses 10 kg and 4 kg and are connected by a spring of negligible mass and placed on a frictionless horizontal surface. An impulse gives a velocity of 14 m/s to the heavier block in the direction of the lighter block. The velocity of the centre of mass is 5K m/s. Then find the value of K.

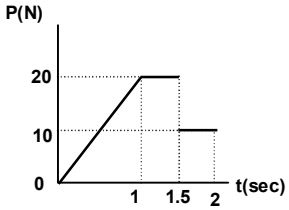
3. The block shown in the figure is in equilibrium. The acceleration of the block just after the string burns is $\frac{ng}{3}$. Find the value of n.



4. The 1/v versus displacement graph of a particle is shown in the figure, where v is the speed of the particle. The particle is moving in a straight line along positive x-axis. Calculate the time taken (in seconds) by the particle to reach from the point A to B.



5. A body of mass 6.25 kg is travelling in a horizontal straight line with a velocity of 3 m/sec when a horizontal force P is applied to it at right angle to the initial direction of motion. If P varies according to the accompanying graph, remains constant in direction and is the only force acting on the body in its plane of motion, find the magnitude of the velocity of the body when t = 2 sec.



space for rough work

SECTION – II : CHEMISTRY

PART – A : (One or more than one Options Correct Type)

This section contains **7 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D), out of which **ONE** or **MORE THAN ONE is correct.**

1. Which of the following is/are correct order of first ionization energies ?
   (a) B > Tl > Ga > Al > In (b) Be > Mg > Ca > Sr > Ba
   (c) B > Al > Ga > In > Tl (d) F > Cl > Br > I

2. Which of the following orbitals are used in the sp^3d^3 hybridisation of IF_7 molecule ?
   (a) $d_{x^2-y^2}$ (b) dz^2 (c) dxy (d) dyz

3. Which of the following set of quantum numbers is/are correct ?
   (a) n = 4, l = 1, m = +1, s = +1/2 (b) n = 3, l = 3, m = +3. s = +1/2
   (c) n = 1, l = 0, m = 0, s = +1/2 (d) n = 4, l = 3, m = -3, s = +1/2

4. Which of the following processes do not involve absorption of energy?
   (a) $S(g) + e^- \longrightarrow S^-(g)$ (b) $O^-(g) + e^- \longrightarrow O^{2-}(g)$
   (c) $Cl(g) + e^- \longrightarrow Cl^-(g)$ (d) $O(g) + e^- \longrightarrow O^-(g)$

5. Which of the following is correctly matched :
   (a) $IO_2F_2^-$ $\rightarrow$ sp^3d Hybridisation of central atom
   (b) F_2SeO $\rightarrow$ sp^3 Hybridisation of central atom
   (c) $ClOF_3$ $\rightarrow$ sp^2 Hybridisation of central atom
   (d) XeF_5^+ $\rightarrow$ $sp^3\,d^2$ Hybridisation of central atom

6. Ammonium carbamate dissociates as

   $$NH_2COONH_4(s) \rightleftharpoons 2NH_3(g) + CO_2(g)$$

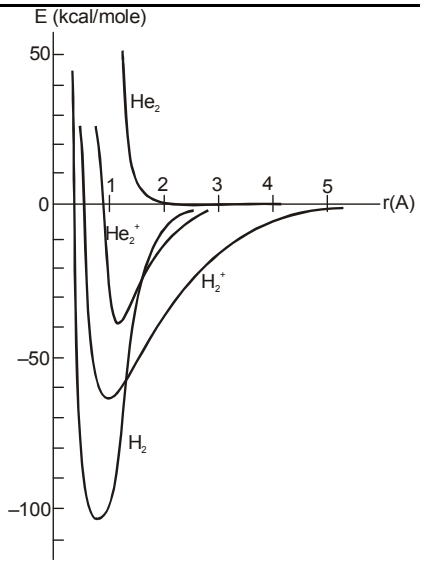
   In a closed vessel containing ammonium carbamate in equilibrium, (pressure at equilibrium of CO_2 is P) ammonia is added such that partial pressure of NH_3 now equals to initial total pressure.
   (A) The total pressure after adding the ammonia is $\frac{31}{9}P$
   (B) Total pressure before adding the ammonia is 3P
   (C) The ratio of total pressure after adding the ammonia and before the adding the ammonia is 31/27.
   (D) The total pressure after adding of ammonia is $\frac{31}{27}P$.

7. For the reaction, $3BrO^- \longrightarrow BrO_3^- + 2Br^-$ in an aqueous alkaline medium at 80°C, the value of the rate constant in the rate law in terms of $\frac{-d[BrO^-]}{dt}$ is 0.056L $mol^{-1}s^{-1}$. What will be the rate constant when the rate law is stated in terms of $\frac{d[BrO_3^-]}{dt}$?
   (A) $18.7\times10^{-3}\,L\,mol^{-1}s^{-1}$ (B) $37.4\times10^{-3}\,L\,mol^{-1}s^{-1}$
   (C) $0.0187\,L\,mol^{-1}s^{-1}$ (D) $18.7\times10^{-2}\,L\,mol^{-1}s^{-1}$

Single Correct Answer Type

This section contains **6 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE is correct.**

8. The following graph is given, between total energy and distance between the two nuclei for species H_2^+, H_2, He_2^+ & He_2, which of the following statements is correct :



(a) He_2^+ is more stable than H_2^+.

(b) Bond dissociation energy of H_2^+ is more than bond dissociation energy of He_2^+.

(c) Since bond orders of He_2^+ and H_2^+ are equal hence both will have equal bond dissociation energy.

(d) Bond length of H_2^+ is less than bond length of H_2.

9. On dissolving moderate amount of sodium metal in liquid NH_3 at low temperature, which one of the following does not occur

(a) Blue coloured solution is obtained (b) Ions are formed in the solution

(c) Liquid becomes good conductor of electricity (d) Liquid ammonia remains diamagnetic

10. If in the equilibrium $CaF_2(s) \rightleftharpoons Ca^{+2} + 2F^-$, the $\left[Ca^{+2}\right]$ is increased 4 times, then the F^- concentration will

(A) increase 4 times

(B) increase 2 times

(C) decrease to half of its original concentration

(D) decrease to one-fourth of its original concentration

11. Following reaction occurs at 25°C,

$2NO(g) + Cl_2(g) \rightleftharpoons 2NOCl$;

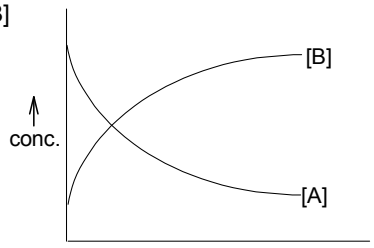
at equilibrium pressure of NO, Cl_2 & NOCl are 1×10^{-5} atm, 1×10^{-2} atm & 1×10^{-2} atm respectively. The value of ΔG^o is

(A) –45.65 kJ (B) –28.53 kJ (C) –22.82 kJ (D) –57.06 kJ

12. For the reaction A → nB, at the point of intersection of 2 curves show, the [B] can be given by (where A_o is initial concentration of A)

(A) $\frac{nA_o}{2}$ (B) $\frac{A_o}{n-1}$

(C) $\frac{nA_o}{n+1}$ (D) $\left[\frac{n-1}{n+1}\right]A_o$



13. The first ionization enthalpies of four consecutive elements presents in the second period of the periodic table are 8.3, 11.3, 14.5 and 13.6 eV, respectively. Which one of the following is the first ionization enthalpy of nitrogen?

(A) 13.6 (B) 14.5 (C) 11.3 (D) 8.3

PART-C
Integer Answer Type

This section contains **5 questions**. The answer to each of the questions is a single digit integer, ranging from **0 to 9**.

1. Number of electrons with m = 0 value in phosphorous atom are

2. Total number of nodal planes in the atomic orbital used in sp^3 hybridisation.

3. Light of wavelength 2000 Å falls on an aluminium surface (work function of Al = 4.2 eV). Therefore, the stopping potential of the experiment in volt unit is:

4. Total number of spectral lines when electron jumps from 8th orbit to 2nd orbit is 20 + n, then n is

5. A certain buffer solution contains equal concentration of X^- & HX. K_b for X^- is 10^{-10}. Calculate pH of buffer.

space for rough work

SECTION – III : MATHEMATICS

PART – A : (One or more than one Options Correct Type)

This section contains **7 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D), out of which **ONE** or **MORE THAN ONE is correct.**

1. Let $I=\int_0^1\sqrt{\frac{1+\sqrt{x}}{1-\sqrt{x}}}dx$ and $J=\int_0^1\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}dx$, then correct statement is

   (A) $I+J=2$ (B) $I-J=\pi$ (C) $I=\frac{2+\pi}{2}$ (D) $J=\frac{4-\pi}{2}$

2. Let the function f(x) be thrice differentiable and satisfies $f(f(x))=1-x$ for all $x\in[0,1]$, If $J=\int_0^1 f(x)dx$ and $f''\left(\frac{4}{5}\right)=0$, then which of the following is(are) true?

   (A) $f\left(\frac{1}{3}\right)+f\left(\frac{2}{3}\right)=1$

   (B) $J=\frac{1}{2}$

   (C) $f''(x)=0$ has at least one root in $x\in\left(\frac{1}{4},\frac{3}{4}\right)$

   (D) f ”(x) = 0 at least one root in $x\in\left(\frac{1}{2},\frac{4}{5}\right)$

3. Suppose that f(x) is a differentiable invertible function, $f(x)\neq 0$ and $h(x)=\int_1^x f(t)dt$. Given that $f(1)=f'(1)=1$ and g(x) is inverse of f(x).,
   Let $G(x) = x^2g(x) – xh(g(x))$ $\forall x\in R$. Which of the following are correct:
   (A) G’(1) = 2 (B) G’(1) = 3 (C) G”(1) = 2 (D) G”(1) = 3

4. Let $f(x)=\sin^{-1}(\cos x)\cos^{-1}(\sin x)\forall x\in[0,2\pi]$, then which of the following statement(s) is/are correct?
   (A) f(x) is continuous and differentiable in $[0,2\pi]$

   (B) Range of f(x) is $\left[-\frac{\pi^2}{4},\frac{\pi^2}{4}\right]$

   (C) $x=\pi$ is a point of global minima as well as local minima

   (D) $\int_0^{\pi} f(x)dx=0$

5. If the sides of a triangle very slightly in such a way that its circumradius remains constant, then $\frac{da}{\cos A}+\frac{db}{\cos B}+\frac{dc}{\cos C}$ is equal to
   (A) 6R (B) 2R
   (C) 0 (D) 2R(dA + dB + dC)

6. If $f(x)=\left(x^2-1\right)^{(n+1)}\left(x^2+x+1\right)$, $n\in N$ and f(x) has a local extrimum at x = 1, then n is equal to
   (A) 2 (B) 3
   (C) 4 (D) 5

7. The points of extrema of $f(x)=\int_0^{x^2}\left(\frac{t^2-5t+4}{2+e^t}\right)dt$ are
   (A) $x=-2$ (B) x = 1
   (C) x = 0 (D) $x=-1$

Single Correct Answer Type

This section contains **6 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE is correct.**

8. If $f : R \to R$ satisfies $f(x + y) = f(x) + f(y)$, for all $x, y \in R$ and $f(1) = 7$, then $\sum_{r=1}^{n} f(r)$ is

(A) $\frac{7n}{2}$ (B) $\frac{7(n+1)}{2}$

(C) $\frac{7n(n+1)}{2}$ (D) $7n$

9. Let $S(x) = \int_{x^2}^{x^3} \ln t\,dt$, $(x > 0)$ and $H(x) = \frac{S'(x)}{x}$, then $H(x)$ is

(A) continuous but not differentiable in its domain
(B) continuous & differentiable in its domain
(C) neither continuous nor differentiable in its domain
(D) None of these

10. The value of the $\int_0^1 \cot^{-1}(1 - x + x^2)dx$ is equal to

(A) $\pi - \ln 2$ (B) $\frac{\pi}{2} - \ln 2$

(C) $\pi + \ln 2$ (D) $\frac{\pi}{2} + \ln 2$

11. The value of $\int x^m (x^{2m} + x^m + 1)(2x^{2m} + 3x^m + 6)^{\frac{1}{m}} dx$ is equal to

(A) $\frac{1}{6(m+1)}(2x^{3m} + 3x^{2m} + 6x^m)^{\frac{(m+1)}{m}}$ (B) $\frac{x^{m+1}}{(m+1)}(2x^{2m} + 3x^m + 6)^{\frac{(m+1)}{m}}$

(C) $\frac{x^{m+1}}{(m+1)}(2x^{2m} + 3x^m + 6)^{\left(\frac{m}{m+1}\right)}$ (D) $\frac{1}{6(m+1)}(2x^{3m} + 3x^{2m} + 6x^m)^{\frac{m}{m+1}}$

12. If $f(x) = \cot^{-1}\frac{3x - x^3}{1 - 3x^2}$ and $g(x) = \cos^{-1}\frac{1 - x^2}{1 + x^2}$, then $\lim_{x \to a} \frac{f(x) - f(a)}{g(x) - g(a)}$, $(0 < a < 1/2)$ is

(A) $\frac{3}{1 + x^2}$ (B) $-\frac{3}{2}$

(C) $\frac{3}{2}$ (D) $\frac{3}{2(1 + a^2)}$

13. The set of points on the curve $xy = 2$, where slope of tangent is positive, is

(A) ϕ (B) $\left\{\left(x, \frac{2}{x}\right) \mid x \in R^+\right\}$

(C) $\left\{\left(x, \frac{2}{x}\right) \mid x \in R^-\right\}$ (D) $\left\{\left(x, \frac{2}{x}\right) \mid x \text{ is any non zero real number}\right\}$

PART-C
Integer Answer Type

This section contains **5 questions**. The answer to each of the questions is a single digit integer, ranging from **0 to 9**.

1. $\int_{-\pi/4}^{\pi/4} \frac{\sec^2 x dx}{1+e^x}$ equals

2. If $\lim_{x\to 0} \frac{ae^x - b\cos x + ce^{-x}}{x\sin x} = 2$, then (a + b + c) =

3. Let f:R$\to$R be defined as $f(x) = \int_0^x (\sin^2(t)+1)dt$ and g is the inverse function of f, then $g'\left(\frac{3\pi}{4}\right) = \frac{\lambda}{2}$, then λ is equal to

4. Let f(x) be a differentiable function on R such that $2f(x+y)+f(x-y) = 3f(x)+3f(y)+2xy$ for all $x, y \in R$. If f'(0) = 0, then find the value of f(3).

5. Let $f(x) = \begin{cases} x(x-1)(x-2) & ; 0 \le x < n \\ \sin(\pi x) & ; n \le x \le 2n \end{cases}$. If $n \in N$ then find the least value of n for which f(x) has more points of minima than maxima in [0, 2n].

space for rough work

FIITJEE COMMON TEST

BATCHES: CPA [A LOT]
PHASE TEST – 1 (PAPER - 1)
ANSWER KEY
SET - A
PHYSICS

1.	**B, C, D**	**2.**	**A, D**	**3.**	**A, B**	**4.**	**D**
5.	**B**	**6.**	**A, C, D**	**7.**	**C**	**8.**	**C**
9.	**A**	**10.**	**B**	**11.**	**C**	**12.**	**A**
13.	**B**						

PART - C

1.	**5**	2.	**2**	3.	**4**	4.	**6**
5.	**5**						

CHEMISTRY

1.	**ABD**	**2.**	**ABCD**	**3.**	**ACD**	**4.**	**ACD**
5.	**ABD**	**6.**	**ABC**	**7.**	**AC**	**8.**	**B**
9.	**D**	**10.**	**C**	**11.**	**A**	**12.**	**C**
13.	**B**						

PART - C

1.	**9**	**2.**	**3**	**3.**	**2**	**4.**	**1**
5.	**4**						

MATHS

1.	**BD**	**2.**	**ABCD**	**3.**	**AD**	**4.**	**BCD**
5.	**CD**	**6.**	**BD**	**7.**	**ABCD**	**8.**	**C**
9.	**B**	**10.**	**B**	**11.**	**A**	**12.**	**B**
13.	**A**						

1.	**1**	**2.**	**4**	**3.**	**1**	**4.**	**9**
5.	**3**						

SOLUTIONS :

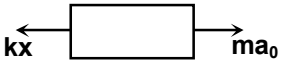
PHYSICS

5. apply work-energy theorem

$$mgz-\left[\frac{1}{2}K\left(\frac{mg}{K}+z\right)^2-\frac{1}{2}K\left(\frac{mg}{K}\right)^2\right]+Fz=0$$

$\Rightarrow z = 2F/K.$

6. Draw the FBD of block in the frame of accelerated frame. In this frame, block has maximum velocity when it is in equilibrium with respect to accelerated frame 0



Using work energy theorem

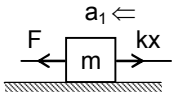
$$\frac{1}{2}kx^2+\frac{1}{2}mv_0^2=ma_0x$$

$$\Rightarrow \frac{1}{2}kx\frac{m^2a_0^2}{k^2}+\frac{1}{2}mv_0^2=\frac{m^2a_0^2}{k}$$

$$\Rightarrow v_0^2=\frac{ma_0^2}{k}$$

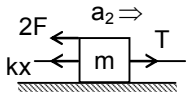
$$\Rightarrow v_0=a_0\sqrt{\frac{m}{k}}$$

7. Let any time t_1 extension in spring is x, so



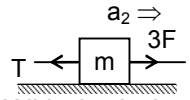
FBD of m

$\Rightarrow$ F – kx = $ma_1 \Rightarrow a_1 = \frac{F-kx}{m}$ ...(1)



FBD of 2m

$\Rightarrow$ T – kx – 2F = 2m a_2 ...(2)



$\Rightarrow$ 3F – T = m a_2 ...(3)

With the help of equations (2) and (3), we have

F – kx = $3ma_2 \Rightarrow \frac{F-kx}{3m}=a_2$ ...(4)

With the help of equations (1) and (4), we have

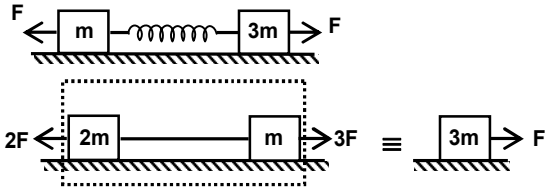
$$\vec{a}_{12}=\vec{a}_1-\vec{a}_2=a_1(-\hat{i})-a_2(\hat{i})=(a_1+a_2)(\hat{i})$$

$$V_{12}\frac{dV_{12}}{dx}=a_{12}=\frac{4(F-kx)}{3m}\Rightarrow\int_0^0 V_{12}dV_{12}=\frac{4}{3m}\int_0^x(F-kx)dx\Rightarrow 0=\frac{4}{3m}\left[Fx-\frac{kx^2}{2}\right]\Rightarrow X_{max}=\frac{2F}{k}$$

Second method: System is equivalent to

Because

Above system behaves as single unit



12. $N=\frac{mv^2}{R}$ .....(A)

$f_{max} = mg$

$\mu N = mg$

$\mu\frac{mv^2}{R} = mg$

$v = \sqrt{\frac{Rg}{\mu}} = \sqrt{\frac{2\times 10}{0.2}} = 10$ m/s

PART – C

1. For collision the relative velocity of A with respect to B should be along the line AB

3. $a = \frac{4}{5}\frac{kx}{m}; \quad \frac{3}{5}kx = mg$

4. Area under graph = $\int \frac{1}{v}dx = \Delta t$

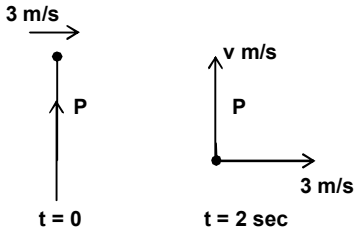
5. Area under the curve = $mv - 0$

$\Rightarrow \quad \left[\frac{1}{2}\times 1\times 20 + 20\times 0.5 + 10\times 0.5\right] = 6.25\, v$

$\Rightarrow \quad [10+10+5] = 6.25\, v$

$\Rightarrow \quad v = \frac{25}{6.25} = 4$ m/s

Speed of body $= \sqrt{3^2+4^2} = 5$ m/s



CHEMISTRY

1. Boron family has abnormal behaviour due to filling of electrons in d- and f-block elements

2. In sp^3d^3 hybridisation, following orbitals are used
S, P_x, P_y, P_z, $d_{x^2-y^2}$, dxy, dz^2

3. acd

4. (a)(c)(d)
Electron gain enthalpy is negative for these.

5. (a)(b)(d)

6. A,B,C

7. A, C

$-\frac{1}{3}\frac{d[BrO^-]}{dt} = \frac{d[BrO_3^-]}{dt}$

$\frac{1}{3}\times 0.056 = \frac{d[BrO_3^-]}{dt}$

8. (b)

9. (d)
Due to free electron liquid ammonia becomes paramagnetic.

10. C
Resulting solution is acidic

11. A

$K_{eq} = \frac{P_{NOCl}^2}{P_{NO}^2 \times P_{Cl_2}} = 10^8$

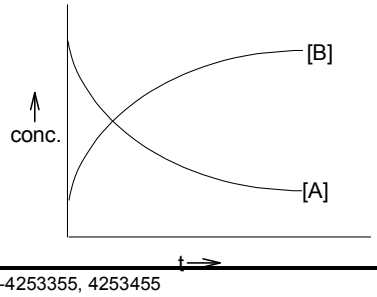
$\therefore \Delta G = -RT\,\ell n\, K_{eq} = -2.303\times 8.314\times 298\times 8 = -45.65$ kJ

12. C

A	$\rightarrow$	n B
A_0		0
$A_0 - \alpha$		$n\alpha$

$A_0 - \alpha = n\alpha$

$\alpha = \frac{A_0}{(n+1)}$



$[B] = n\alpha = \frac{nA_0}{n+1}$

13. B

1. Nine electrons are present
2. 3
3. 2
4. 1
5. 4

MATHS

1. $I + J = \int_0^1 \frac{2}{\sqrt{1-x}}\,dx = 4$

$I - J = \int_0^1 \frac{2\sqrt{x}}{\sqrt{1-x}}\,dx = I_1(\text{say})$ .....(1)

$I_1 = \int_0^1 \frac{2\sqrt{x}}{\sqrt{1-x}} = \int_0^1 \frac{2\sqrt{1-x}}{\sqrt{x}}$ .....(2)

Adding equation (i) & (ii) ;

$2I_1 = \int_0^1 2\left(\frac{x+1-x}{\sqrt{x(1-x)}}\right)dx = 2\int_0^1 \frac{dx}{\sqrt{x(1-x)}}$

$I_1 = \Pi$

2. $f(f(x) = 1 - x$

Replace $x \to f(x)$

$f(f(f(x))+f(x) \Rightarrow 1 \Rightarrow f(1-x) + f(x) = 1$ …(1)

Put x = 1/3, we get f(1/3) + f(2/3) = 1

$J = \int_0^1 f(x)dx = \int_0^1 f(1-x)dx$

$2J = \int_0^1 f(x)dx + \int_0^1 f(1-x)dx = \int_0^1 1.dx$

I = 1/2

From equation (1)

f'(x) = f'(1–x)

for $x \in (1/4, 3/4)$; f'(1/4) = f' (3/4)

Hence, f" (x) = 0 has at least one root in $x \in (1/4, 3/4)$

Also, f"(x) = –f" (1–x)

at x = 1/2 ; f"(1/2) = – f"(1/2) ; f"(1/2) = 0

at x = 4/5 ; f"(4/5) =0 (given)

Hence, f"(x) = 0 has at least one root in $x \in (1/2, 4/5)$

3. $h'(x) = f(x) \Rightarrow h''(x) = f'(x)$

h(1) = 0, f(1) = f'(1) = h'(1) = h"(1) = g(1)

f(g(x)) = x

f' (g(x)). g'(x) = 1

f' (g(1)). g'(1) = 1 $\Rightarrow$ g' (1) = 1

$G(x) = x^2.g(x) – x.\ h(g(x))$

$G'(x) = x^2\ g'(x) + 2x.\ g(x) – h(g(x)) – x.\ h'(g(x)).\ g'(x)$

$= 2x.g(x) – h(g(x))$

G"(x) = 2.g(x) + x.g'(x)

G'(1) = 2g(1) – h (g(1)) = 2g(1) – h(1) = 2

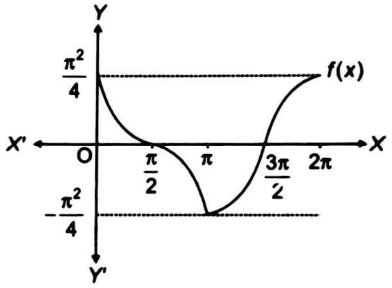
G"(1) = 2.g(1) + g'(1) = 3

4. (BCD)

$f(x) = \sin^{-1}(\cos x).\ \cos^{-1}(\sin x)$

$= \left(\frac{\pi}{2} - \cos^{-1}(\cos x)\right).\left(\frac{\pi}{2} - \sin^{-1}(\sin x)\right)$

$$f(x)=\begin{cases}\left(\frac{\pi}{2}-x\right)^2{}', & 0\le x<\frac{\pi}{2}\\ -\left(\frac{\pi}{2}-x\right)^2, & \frac{\pi}{2}\le x<\pi\\ \left(x-\frac{3\pi}{2}\right).\left(x-\frac{\pi}{2}\right), & \pi\le x<3\pi/2\\ -\left(x-\frac{3\pi}{2}\right).\left(x-\frac{5\pi}{2}\right), & \frac{3\pi}{2}\le x\le 2\pi\end{cases}$$



5. **C, D**

$da=2R\cos A, db=2R\cos BdB$

$dC=2R\cos cdC$

$\therefore \frac{da}{\cos A}+\frac{db}{\cos B}+\frac{dc}{\cos C}=2R(dA+dB+dC)$

Also A + B + C = π

$\Rightarrow dA+dB+dC=0$

$\therefore \frac{da}{\cos A}+\frac{db}{\cos B}+\frac{dc}{\cos C}=0$

6. **B, D**

$f(x)=(x^2-1)^{(n+1)}(x^2+x+1)$

$\Rightarrow f'(x)=(n+1)(x^2-1)^n(2x)(x^2+x+1)+(x^2-1)^{n+1}(2x+1)$

$\Rightarrow f'(x)=(x^2-1)^n\left[2x(n+1)(x^2+x+1)+(x^2-1)(2x+1)\right]$

Clearly, the expression within bracket will always be +ve in the neighbourhood of x = 1.

$\therefore$ the sign of $f'(x)$ depends upon $(x^2-1)^n$

Also, for local extrimum at x = 1

$f'(1^+)$ & $f'(1^-)$ must be of opposite sign

$\therefore$ n must be odd.

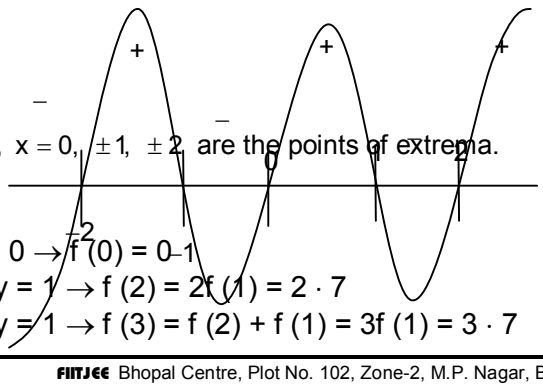
7. **A, B, C, D**

$f'(x)=\frac{(x^4-5x^2+4)}{(2+e^{x^2})}\cdot 2x$

$f'(x)=0 \Rightarrow x(x^2-1)(x^2-4)=0$

$\Rightarrow x=0,\ \pm1,\ \pm2$

$f'(x)=\frac{2}{(e^{x^2}+1)}x(x+1)(x-1)(x+2)(x-2)$



Clearly, $x=0, \pm1, \pm2$ are the points of extrema.

8. $x = y = 0 \rightarrow f(0) = 0$

$x = 1, y = 1 \rightarrow f(2) = 2f(1) = 2 \cdot 7$

$x = 2, y = 1 \rightarrow f(3) = f(2) + f(1) = 3f(1) = 3 \cdot 7$

$x = 2, y = 2 \rightarrow f(4) = f(2) + f(2) = 4f(1) = 4 \cdot 7$ and so on
Hence, $f(r) = 7r$

$$\Rightarrow \sum_{r=1}^{n} f(r) = 7\sum_{r=1}^{n} r = \frac{7n(n+1)}{2}.$$

9. **B**

$$S(x) = \ln x^3 \cdot 3x^2 - \ln x^2 \cdot 2x$$
$$= (9x^2 - 4x)\ln x$$
$$\Rightarrow H(x) = (9x - 4)\ln x$$

$\therefore$ We can clearly say that H(x) is continuous & differentiable in its domain which is $(0, \infty)$.

10. **B**

$$I = \int_0^1 \cot^{-1}(1 - x + x^2)\,dx = \int_0^1 \tan^{-1}\left(\frac{1}{(1 - x(1-x))}\right)dx$$
$$= \int_0^1 \left(\tan^{-1} x + \tan^{-1}(1-x)\right)dx$$
$$= 2\int_0^1 \tan^{-1} x\,dx = 2\int_0^1 \tan^{-1}_{I} x \cdot \underset{II}{1}\,dx$$
$$= 2\left\{\left[\tan^{-1} x \cdot x\right]_0^1 - \int_0^1 \frac{1}{(1+x^2)} \cdot x\,dx\right.$$
$$= 2\left[\frac{\pi}{4} - \frac{1}{2}\ln 2\right] = \frac{\pi}{2} - \ln 2$$

11. **A**

$$I = \int x^m \left(x^{2m} + x^m + 1\right)\left(2x^{2m} + 3x^m + 6\right)^{\frac{1}{m}}\,dx$$
$$= \int x^{m-1}\left(x^{2m} + x^m + 1\right)\left(2x^{3m} + 3x^{2m} + 6x^m\right)^{\frac{1}{m}}\,dx$$

Put, $\left(2x^{3m} + 3x^{2m} + 6x^m\right) = t$

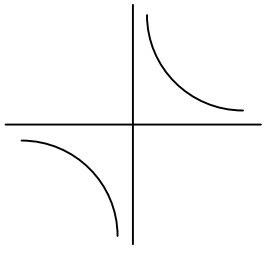
$$\Rightarrow 6m\left(x^{3m-1} + x^{2m-1} + x^{m-1}\right)dx = dt$$

$$\therefore I = \frac{1}{6m}\int t^{\frac{1}{m}}\,dt = \frac{1}{6m} \cdot \frac{t^{\left(\frac{1}{m}+1\right)}}{\left(\frac{1}{m}+1\right)} + c$$
$$= \frac{1}{6(m+1)}\left(2x^{3m} + 3x^{2m} + 6x^m\right)^{\left(\frac{m+1}{m}\right)} + c$$

12. B

13. $y = \frac{2}{x}$

$\frac{dy}{dx} = \frac{-2}{x^2}$ always negative never positive thus ϕ



PART - C

1. 1

$$\int_{-\pi/4}^{\pi/4} \frac{\sec^2 x}{1+e^x}dx = \frac{1}{2}\int \frac{\left(1+e^x\right)+\left(1-e^x\right)}{\left(1+e^x\right)}\cdot \sec^2 x dx = \frac{1}{2}\int_{-\pi/4}^{\pi/4} \sec^2 x dx + \frac{1}{2}\int_{-\pi/4}^{\pi/4} \frac{1-e^x}{1+e^x}\sec^2 x dx$$

$$= \frac{1}{2}\left(\tan x\right)_{-\pi/4}^{\pi/4} + 0\left(\because f(-x) = \frac{1-e^{-x}}{1+e^{-x}}\sec^2(-x) = -\frac{1-e^x}{1+e^x}\sec^2 x = -f(x)\right) \Rightarrow \quad \frac{1}{2}\cdot 2 = 1$$

2. 4

$$\lim_{x\to 0}\frac{ae^x - b\cos x + ce^{-x}}{x\sin x} = 2 \quad a - b + c = 0 \qquad \ldots\ldots (1)$$

$$\lim_{x\to 0}\frac{ae^x + b\sin x - ce^{-x}}{x\cos x + \sin x} = 2 \quad a - c = 0 \qquad \ldots\ldots(2)$$

$$\lim_{x\to 0}\frac{ae^x + b\cos x + ce^{-x}}{-x\sin x + \cos x + \cos x} = 2 \qquad \frac{a+b+c}{2} = 2$$

$a + b + c = 4$ ….(3)

$a–b + c = 0$

$2b = 4$ $\quad b = 2$

$a+c = 2$

$a–c = 0$

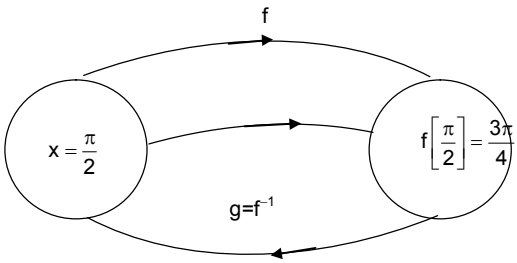
$2a = 2$

$a = 1$ $\quad c = 1$

3. 1

Here, $f'(x) = \sin^2(x) + 1$

So, $g'\left(\frac{3\pi}{4}\right) = \frac{1}{f'\left(\frac{\pi}{2}\right)} = \frac{1}{2}$



4. (9)

$2f(x+y) + f(x-y) = 3f(x) + 3f(y) + 2xy$

$f(0) = 0$

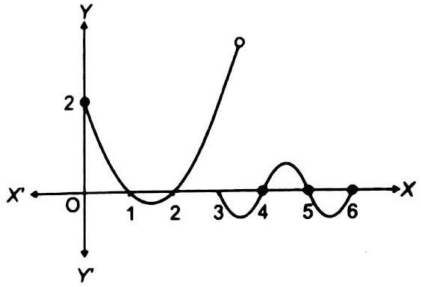
$f'(x) = \lim_{h\to 0}\frac{2f(x+h) - 2f(x)}{2h}$

$f'(x) = \lim_{h\to 0}\left(\frac{3f(x) + 3f(h) + 2xh - f(x-h) - 2f(x)}{2h}\right)$

$= \lim_{h\to 0}\frac{f(x) - f(x-h)}{2(x-(x-h))} + \frac{3}{2}f'(0) + x$

$f'(x) = \frac{f'(x)}{2} + x \Rightarrow f'(x) = 2x \Rightarrow f(x) = x^2$

5. (3) $f(x) = \begin{cases} x(x-1)(x-2); & 0 \le x < n \\ \sin(\pi x); & n \le x \le 2n \end{cases}$



No. of maxima = 2

No. of minima = 3